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Abstract

A four-parameter approximating formula, $R = A \bullet \exp(a) \bullet x^b \bullet f(x)$, accurately represents the skyshine line beam response function (LBRF) as a function of the distance (x) of the source-to-detector separation. Here, A is a constant for a given source energy and $f(x) = \exp(cx) \bullet x^{dx}$ is a damping factor. The four parameters are obtained as follows.

1. The value of parameter a corresponds to that of the LBRF at $x = 1$ meter, which is the result of integrating the basic dose spectrum due to a single scattering particle from an emitted beam for a specified angle and a specified source energy.
2. The value of parameter b corresponds to the slope of a straight line of the response function, $\log R$ vs. $\log x$, in the range of small distance from a source, where a single scattering particle dominates.
3. The damping factor ($f(x)$) represents the attenuation trend of the LBRF at distances far from the source; the values of parameters c and d control the quantity of attenuation.

The necessary reference LBRF data for point mono-directional photon source energies ranging from 0.1 to 10 MeV were generated using the EGS4 Monte Carlo code at 20 emission angles from 0.0 to 180 deg for 24 source-detector distances up to 2000 m. The validity of using the four-parameter formula to interpolate the LBRF in the source-to-detector distance, in the emitted angle, and in the energy was also ascertained. Furthermore, this formula was applied to the skyshine conical beam response function (CBRF) for a neutron and an associated secondary gamma-ray with the source energy ranged from thermal to 3 GeV. It was ascertained that the CBRF could be accurately approximated by an interpolation of the fitting parameters at an arbitrary distance and emitted cosine angle.

I. Introduction

The radiation skyshine problem can be accurately calculated by deterministic methods or Monte Carlo methods. However, their use requires immense computations, which is unsuitable for routine skyshine calculations, or preliminary shield-design analyses. Consequently, researchers have made considerable efforts to develop simple skyshine calculation methods. Among them, there are the integral line beam response method for a collimated point source with an arbitrary energy and an arbitrary emitted angle, the integral conical beam response method for an azimuthally symmetric source, and other approximation methods.

The skyshine dose ($D(x)$) at a distance x from a bare collimated point source that emits $S(E,\Omega)$ photons with an arbitrary energy and angular distribution is

$$D(x) = \int_0^\infty dE' \int_{\Omega_s} d\Omega \bullet S(E',\Omega) \bullet R(E',\Phi,x),$$

where $R(E,\Phi,x)$ is the line beam response function (LBRF), and the angular integration is over all emission directions $(\Omega\, s)$ allowed by source collimation. Here, Φ presents the angle between a photon direction emitted from a point source and the source-to-detector axis, and the cosine of the angle of emission (Φ) is the dot product of the unit vector in the emission direction and a unit vector along the source-to-detector axis.

Lynch et al.[1] calculated the air dose for gamma-rays using a customized Monte Carlo code. The calculations were performed at source-detector separation distances from 1.524 to 30.48 m for source energies from 0.6 to 12 MeV and beam angles with respect to the source-detector axis from 1 to 180 deg. Their results were used as the first line beam response function (LBRF).

On the other hand, an attempted was made to arrange the LBRF results of Lynch et al. to make an approximation by a single-scattering calculation. Trubey[2] calculated the single-scattered flux or dose without any exponential attenuation or buildup factor in air, and showed that his results were valid to about 60 deg for a near-field calculation, where the detector is near the source. Based on Trubey's work, Kitazume[3] modified the single-scatter approximation by incorporating exponential attenuation and a Taylor-type buildup factor. The inclusion of exponential attenuation for both uncollided photons and scattered photons, and that of the consideration of secondary radiation buildup allows this single-scatter method to be applied to more complicated skyshine geometries at far field from the source.

Radiation Research Associates (RRA) carried out more extensive Monte Carlo calculations[4] and single-scattered point kernel calculations[5] for obtaining the LBRF data. Those data were implemented the LBRF method in the SKYSHINE series of codes.[6-8] These response functions were represented using an equation of the $\exp(a-cx)\bullet x^{b}$ form for a fixed value of energy and emitted angle, where three parameters were determined by a least-squares fitting. The maximum deviation and average errors of the LBRF approximated by this formula from reference data were more than 20% on decreasing the source energy. Small sporadic discontinuities were seen in the fit parameters of the LBRF in SKYSHINE-III. Furthermore, a deficiency was apparent in the extrapolated LBRF related to the energy or the distance, using fitting parameters.[5] Therefore, it was noted[9] that to calculate the LBRF at a neighboring angle or energy given in tables, an approximate LBRF should be obtained by interpolation between the values of the LBRFs at the tabulated angles or energies, and not interpolation of the fit parameters.

The authors[10] have proposed a four-parameter approximation formula, $R(E,\Phi,x)=\Re Ek\bullet\exp(a+cx)\bullet x^{b+dx}$. By adding x^{dx}, the maximum deviations of the LBRF become smaller than 10% over a wide energy range. Shultis and Faw[8] stated that the four-parameter formula is superior to the three-parameter formula for energies of less than a few MeV; but it does not approximate the LBRF data for energies above 10 MeV as well as the three-parameter formula does. Where the LBRF data for energies above 10MeV were calculated by Brockhoff et al..[9] The values of the fit parameters (*a, b, c,* and *d*) in SKYSHINE III were chosen by a least-squares procedure for the logarithmic plot of the LBRF.

The success of a four-parameter fit in the skyshine LBRF is due to not only the monotonic and slow variation of the photon interaction cross section for air with energy, but also because the distances of one mean free path over a wide energy range are sufficiently longer. This is easy to understand from the fact that **Figs. 29** to **34** of Ref. 1 (tissue doses rates in air as a function of the source-detector separation distance) show straight lines having the same slope independently of the source energies and the emitted angles. Therefore, the behavior of LBRF at a distance near the source is well explained by the single-scatter approximation. On the other hand, the multiple-scattered gamma-rays make significant contributions to attenuation of the LBRF at distances far from the source. For low-energy gamma-rays incident on a light material, the Compton scattering is predominant in the total cross section. As the incident energy decreases, the energy loss of gamma-rays due to Compton scattering decreases proportionally, while gamma-rays with back scattering proportionally increase. Consequently, gamma-rays with almost the same energy as the incident ones must undergo repeat multiple scattering before they are absorbed.[11] At a distance sufficiently separated from the source, a peak of the specified material appears in the energy spectrum, for example a peak at around 60 keV for water,[12] because the photoelectric interaction dominates over the Compton scattering. At a distance far from the source, the peak of a specified material begins to cause the attenuation of the LBRF. This behavior causes the attenuation of $[\log f(x)]/x$ to become a linear function.

In this paper, it is clarified that the LBRF at a distance near the source is greatly dependent on single-scattered photons, and that at a distance far from the source, the LBRF is dependent on multiple-scattered photons. These dependencies are reflected on the determination of parameters in the four-parameter formula. Therefore, the four-parameter formula is shown to well reproduce the LBRF for gamma-rays. Furthermore, each parameter is adjusted to vary monotonically and smoothly with regard to the emitted angle (Φ) and the logarithm of the source energy (E). As a result, to calculate the LBRF at a neighboring angle or energy, given in tables, the approximate LBRF is obtained by interpolation of the fit parameters of the four-parameter empirical formula.

Another useful skyshine response function involves the conical beam response function (CBRF). Gui et al.[13] explained in detail several approximating formulas[6, 11-18] of the LBRF and the CBRF for neutrons and secondary gamma-rays. However, the neutron energy was limited to 14 MeV in their work. In the present paper, the high-energy CBRF and its fitting parameters using the four-parameter formula are prepared as described below.

The SHINE-III[19] code is now being developed by JAERI (Japan Atomic Energy Research Institute) and KEK (High Energy Accelerator Research Organization) for the J-PARC project: the construction of high-energy accelerators and target facilities. The SHINE-II code[20] using a semi-empirical formula[17] is already available for evaluating the neutron and secondary gamma-ray skyshine doses. However, the CBRF data are limited to 400 MeV. For the SHINE-III code, the energy and distance range of the CBRF calculations[19] have been extended using the NMTC/JAERI97[21] and MCNP4A[22] code systems. In order to accurately reproduce the values of the CBRF at a distance near the source, the four-parameter formula was used in an attempt to fit the CBRF.[19]

II. Preparation of Reference Data of the LBRF and the CBRF

Before discussing an approximating formula for the LBRF and the CBRF, it is first necessary to obtain reference data (R) of LBRF at different distances from the source for each beam energy and direction. The necessary reference LBRF data[23] were calculated with the EGS4 code[24] using photon cross sections taken from the PHOTX data library.[25] A surface crossing estimator of a disk of radius $r = 0.1 \bullet x$ was located on the source-to-detector axis vertically, where x is the source-detector separation distance. When the emitted angle (Φ) is small and $\cos(90° - \Phi)$ is less than or equal to 0.1, the radius is set to $0.8 \bullet \cos(90° - \Phi)$.

The LBRF was calculated at 24 distances (x), at 11 discrete energies (E), and for 20 discrete beam directions (Φ). The source-detector distances were 10, 20, 50, 80, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, 1400, 1500, 1600, 1700, 1800, 1900, and 2000 m. The discrete energies selected were 0.1, 0.2, 0.5, 0.7, 1, 2, 3, 4, 5, 7, and 10 MeV, and the emitted angles were 0.0, 0.1, 1, 2.5, 5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 140, 160, and 180 deg.

In order to use a response function for the SHINE-III code,[19] the CBRF for neutrons must include the energy range to 3 GeV and the distance range to 2000 m. The NMTC/JAERI97[21] was used to calculate the neutron and secondary gamma-ray skyshine dose distributions. This code is a nucleon-meson transport Monte Carlo code using an inter-nuclear cascade model and preequiblium model, and it was used for neutron calculation which energy is down to 20 MeV. The energy and directional dependence of neutrons below 20 MeV were stored to external file, and these neutrons are treated as sources for successive MCNP4A[22] calculation down to thermal energy. The MCNP4A code was used to calculate the neutron and secondary gamma-ray skyshine dose distributions, where the neutron cross section was taken from the JENDL-3.2 library.[26] The CBRF was evaluated at 20 distances (>10 MeV) or 32 distances (≤ 10 MeV) using scoring cells centered about the source-detector distances. The angular intervals of upward emission cosine angles were 0.0-0.2 (0.1), 0.2-0.4 (0.3), 0.4-0.6 (0.5), 0.6-0.8 (0.7), and 0.8-1.0 (0.9). The related neutron source energies (E) selected were 3, 2, 1.5, 1.2, 1, 0.8, 0.6, 0.4, 0.1 GeV; 50, 10, 5, 1, 0.1 MeV; 1 keV; 10 eV, and thermal.

III. Physical Meaning of Parameters

1. A Four-Parameter Empirical Formula for the LBRF

The integral line beam skyshine method is based on the availability of a line beam response function ($R(E, \Phi, x)$),[5, 6-8] which gives the air kerma(Gy per photon) at distance x from a point source emitting photons of energy E into an infinite air medium at an angle Φ relative to the source-to-detector axis (the air-ground interface is neglected in this case). The LBRF for fixed values of E and Φ can be fit to the following four-parameter empirical formula[10] over a large range of x:

$$R(E,\Phi,x) = \Re Ek\,(r/r_0)^2 \bullet \exp(a + cxr/r_0) \bullet [x(r/r_0)]^{b+dxr/r_0}, \qquad (1)$$

where r is the actuary air density in the same units as the reference density, $r_0 = 0.001225$ g/cm^3. The term $(r/r_0)^2$ is a correction factor introduced by Zerby[27] to adjust the dose rate in air at a given reference density (r_0) to the corresponding value at the actuary air density (r). E is measured in MeV and x in meters, $\Re$ is the Gy/(dose response), and the constant (k) is equal to 1.308 x 10^{-11}/MeV/photon. The fit parameters (a, b, c, and d) depend on the photon energy and the source emission angle. In the following $r = r_0$ is assumed.

2. Relationship Between Two Parameters (a and b) and LBRF Near the Source

The variations of the LBRFs (unit in tissue dose rate) were shown for a specified E and a specified emitted angle (Φ) in **Figs. 29** through **34** of Ref.1. The values of the function $\log R$ versus $\log x$ are seen to fall on a straight line having the same slope, b. The value of b is almost equal to −1, and has no relationship with the magnitude of the energy (E) and the emitted angle (Φ).

As an example, LBRFs of 1 MeV for emitted angles of $\Phi =$ **1**, 15, 30, 60, 90, 135, and 180 degrees were calculated at distances of 1 to 180 m from the source using the EGS4 code. These are compared with those of Lynch et al.[1] in **Fig. 1**. The results are in good agreement up to 30 m, but the values of LBRF calculated by EGS4 deviate from linearity beyond 30 m. When distance x is small, the value of the slope of the straight line corresponds to the parameter b of the right-hand term in Eq. (1). That is, the value of the parameter b is obtained using the LBRF at distances of x_1 and x_2 within 30 m,

$$b = [\log R(x_2) - \log R(x_1)]/(\log x_2 - \log x_1). \qquad (2)$$

In the case of x = 1, the LBRF in Eq. (1) is

$$R(1) = \Re Ek \bullet \exp(a + c), \qquad (3)$$

$$\approx \Re Ek \bullet \exp(a), \qquad (3)'$$

where $|a| >> |c|$.

The logarithm of $R(1)$ is given using the value of b in Eq. (2), as follows:

$$\log R(1) = \log R(x_2) + b \bullet \log(x_2). \qquad (4)$$

The parameter a is represented from Eq. (3)' as follows:

$$a = \ln\left[R(1)/(\Re Ek)\right]. \qquad (5)$$

Now, the values of parameters a and b were determined using Eqs. (5) and (2). The reference LBRF data start from 10 m, followed by 20 m. The LBRFs at 10 and 20 m were used to determine the value of b. The values of a and b are listed in **Table 1**, for emitted angles of 1, 30, 60, 90, 135, and 180 deg for a source energy of 1 MeV. The values of b are '−1' within 7%. The straight lines in **Fig. 1** were drawn using the values of a and b given in **Table 1.**

Trubey[2)] gave the following approximated equation to present the skyshine dose rates with a form of only single-scattering contribution, without any exponential attenuation or buildup factor in air:

$$R(E,\Phi,x) = \Re N/(x \bullet \sin\Phi)\int_{\Phi}^{p} ds\ /d\Omega(E,q)dq. \qquad (6)$$

Here, N is electron density (cm^{-3}). The fraction $1/x$ of Eq. (6) corresponds to $b = -1$. On the other hand, the ratios of the singly scattered gamma-ray flux to the total flux as a function of the angle of emission of the source energy (1 MeV) were plotted in Fig. 23 of Ref. 1. This figure showed that the ratios decreased with increasing angle and distance from the source. The ratios at a distance of 19.812 meters were 70% for 30 deg, 50% for 60 deg, and 40% for 90 deg. The problem of underestimating the LBRF by single-scattered gamma-rays was remedied by the following procedures: Trubey introduced a simple formula that neglected any exponential attenuation, and Kitazume[3)] introduced a formula that considered both exponential attenuation and buildup effects of scattered photons. Trubey's procedure was effective for the LBRF up to 30.48 meters. However, the LBRFs drop below the straight lines at distances greater than 40 m from the source (Fig.1). In order to fit to such behavior, an even greater damping factor having new two parameters, c and d, must be added to the single scattering formula.[10)]

3. Relationship Between Parameters (c and d) and LBRF at a Distance Far from the Source

The following damping factor is introduced.

$$f(x) = \exp(cx) \bullet x^{dx}. \qquad (7)$$

The LBRF, $R(x)$, of Eq. (1) is represented by

$$R(x) = \Re Ek \bullet \exp(a) \bullet x^b \bullet f(x). \qquad (8)$$

The function $f(x)$ is

$$f(x) = R(x) / \left[\Re Ek \bullet \exp(a) \bullet x^b\right], \qquad (9)$$

and the logarithm of the function $f(x)$ is

$$\log f(x) = x\left[c \bullet \log e + d \bullet \log x\right]. \qquad (9)'$$

The values of $\log f(x)$ are given from the reference LBRFs, $R(x)$, and the parameters a and b, determined by Eqs. (5) and (2). Furthermore, $\left[\log f(x)\right]/x$ is a linear function of $\log x$:

$$\left[\log f(x)\right]/x = c \bullet \log e + d \bullet \log x \qquad (10)$$

The values of $\left[\log f(x)\right]/x$ for 1 MeV are plotted in **Fig.2** at emitted angles of Φ = 2.5, 30, 60, 90, and 140 deg. These fall on a straight line as well at distances greater than 300 m. The value of c in Eq. (10) corresponds to $\left[\log f(1)\right]/\log e$ at $x = 1$, but the value of c should be seen as a parameter depending on the various slopes of straight lines for $\left[\log f(x)\right]/x$ at a distance far from the source. The value of d is the straight line slope and becomes steep with increasing emitted angle (Φ). That is, the value of d decreases with increasing Φ. This suggests that the magnitude of d leads to the attenuation of LBRF at distance far from the source.

The magnitude of c encourages or alleviates the attenuation of the LBRF at a distance far from the source. The value of c is negative for an emitted angle of Φ = 2.5 deg, and positive for emitted angles of Φ = 30, 60, 90, and 140 deg. The values of c for 90 and 140 deg are almost the same. There are a few data of $\left[\log f(x)\right]/x$ which deviate from the line at distances of less than 300 m. Under such cases, the accuracy of LBRF, approximated by Eq. (1), is not affected by the values of parameters c and d.

4. Approximation of the LBRF by the Four-Parameter Empirical Formula

The above four-parameter empirical formula was fitted to the LBRF calculated by the EGS4. To explicitly separate the principal energy dependence of the response function, the LBRF is approximated as

$$R(E, \Phi, x) / (\Re Ek) = \exp(a + cx) \bullet x^{b+dx}.$$

The fitting parameters (a, b, c, and d) can be obtained for a given beam energy and direction by fitting the approximating function to values of the LBRF calculated at a set of discrete source-to-detector distance of x_m, $m = 1, \cdots, M$. i.e., to $R(E, \Phi, x_m) = R_m$. As a criterion to fit the four-parameter formula to R_m, the "minimization of maximum fractional deviation" was selected in this study. The first step was to choose the values of parameters a and b using Eqs. (5) and (2). As the second step, the values of parameters c and d were chosen from Eq. (10) by minimization of maximum fractional deviation; $\log f(x)$ was obtained from Eq. (9) using the values of fixed a and b chosen by the first step. The values of the fitting parameters were obtained by fits to Monte Carlo data that must have contained statistical errors. Furthermore, the values of a and b were determined by the LBRF data at a distance near the source, and those of c, and d were determined by the LBRF data at a distance far from the source. The values of the four parameters have a good chance of having more than one set of values by fitting to the LBRFs with additional reference data at the middle distance between the source and the detector.

Then, as the third step, the values of a, b, c, and d were chosen so that the maximum fractional deviation could be adjusted to be as small as possible, and the value of each parameter changed monotonically and smoothly as a function of the emitted angle and the source energy. The third step is especially important to approximate the LBRF by interpolation of the fit parameters.

Between the first and third steps, the maximum deviation of parameter a is only 0.74%, but that of b is 6.3%.

Although a simultaneous four-parameter least-squares fit is easy, small sporadic discontinuities were seen in the fit parameters of the LBRF in SKYSHINE-III[8]. Furthermore, there are cases that the obtained values violate the rules that the values of the parameters a and b represent the behavior of attenuation of the LBRF at a distance near the source, and those of parameters c and d represent the behavior of attenuation of the LBRF at a distance far from the source. As a result, the four parameters obtained by the former method do not give a close agreement between the approximate LBRF and the calculated LBRF data for a large emitted angle and for source energies above 10 MeV.

As an example, the values of the four-parameter for a source energy of 1 MeV are listed in **Table 2**, where columns 6 and 7 give the root-mean-square deviation. Here, the fitting range of the LBRF is limited to the distance within which the statistical error of the EGS4 calculation does not go over 10%. With the three step fit procedure, the approximate formula reproduced the reference LBRF of 10 MeV calculated by the EGS4 as well as LBRFs for a few MeV.

IV. Relationships of Four Parameters with Emitted Angle and Source Energy

The values of the four parameters for a specified source energy were determined following the process mentioned in section III.4. The values of each parameter are plotted as a function of the emitted angle in **Fig. 3**. The maximum deviation from all LBRFs is almost within 10%.

1. Relationship between Parameter a and the Emitted Angle

The value of parameter a is determined by Eq. (5). The values of parameter a are dependent on the emitted angle and give decreasing curves in the region below 90 deg, and flat curves in the region above 90 deg. The behaviors of the parameter a are the same as those of $R(1)$. The value of a for a specified angle decreases with increasing source energy. This is due to the decrease in the Compton scattering cross section with increasing energy.

2. Relationship Between Parameter b and the Emitted Angle

The value of parameter b is determined by Eq. (2). The values of the parameter b, dependent on emitted angle, give increasing curves in the region below 30 deg (the first region), a gently increasing curve in the region between 30 and 90 deg (the second region), and a flat curve in the region above 90 deg (the third region).

The equation of a single-scattering calculation proposed by Trubey was without any exponential attenuation or buildup factor. In such a case, the value of b is -1. However, by considering the exponential attenuation of un-scattered and single scattered photons, the value of b becomes smaller than -1 below a source energy of 3 MeV. Next, with an increasing contribution of multiple-scattering photons, the value of b increases with increasing emitted angle, and the behaviors of the spectra in the first and second regions are explained. The behaviors of the emitted angular spectra in the third region are caused by the dominance of multiple-scattered photons. At 10 MeV, the differences of b below 100 deg are due to the contribution of bremsstrahlung radiation. The values of b increase with increasing source energy because the attenuation cross section decreases with increasing source energy.

3. Relationship Between Parameters c and d and the Emitted Angle

The values of parameters c and d of damping factor $f(x)$ are determined from Eq. (10), at distances far from the source. As shown in Fig. 3, the behaviors of parameter d dependent on the emitted angle in the region below 90 deg are decreasing curves with increasing Φ, and those in the region above 90 deg are gently decreasing curves. The curves of d show a similar shape for the various source energies, and d decreases with decreasing energy. Such trends of the d parameter correspond to the circumstance of the attenuation of the LBRFs at a distance far from the source. In the cases of a large emitted angle, the energy of a single scattered photon becomes lower even for a higher source energy, and multiple-scattered gamma-rays make significant contributions to the attenuation of the LBRF at distances far from the source. For low-energy gamma-rays incident on a light material, the Compton scattering is predominant in the total cross section. As the incident energy decreases, the energy loss of gamma-rays due to the Compton scattering decreases proportionally, while gamma-rays with back scattering proportionally increase. Consequently, gamma-rays with almost the same energy as the incident ones must undergo repeat multiple scattering before they are absorbed.[12] At a distance sufficiently separated from the source, the peak of a specified material appears in the energy spectrum; as an example, a peak appears at around 60 keV for water[12], because the photoelectric interaction dominates over the Compton scattering. At a distance far from the source, the peak of a specified material begins to cause the attenuation of the LBRF. This behavior causes the attenuation of $[\log f(x)]/x$ to become a linear function.

The decrease of d becomes looser beyond Φ = 90 deg. For angles above 90 deg, the contribution to the LBRF by the multiple-scattered gamma-rays are almost independent of the emitted angle.

The values of c are negative for the source energy above 5 MeV, and are positive below 3 MeV, except at small angles. At Φ= 40 deg, the value of c is zero for 3 MeV, as can be seen in **Fig. 3**. The attenuation curves for seven source energies at Φ= 40 deg are plotted in **Fig. 4**. The attenuation curve for 3 MeV is approximated by $\Re Ek \bullet \exp(a) \bullet x^{b+dx}$. The factor x^{b+dx} brings about the rapid attenuation at a distance far from the source, because the sign of $b+dx$ is negative. The attenuation accelerates with decreasing energy. On the other side, the exponent (cx) pushes up the attenuation curve, x^{b+dx}, in the case of $c > 0$ (below 3 MeV), and alleviates the rapid attenuation. In the case of $c < 0$ (above 5 MeV or for a small angle, below 3 MeV), the exponent (cx) lowers the attenuation curve, x^{b+dx}, and encourages attenuation.

At 10 MeV, the differences in the parameters of c and d between 40 and 100 degrees are due to the contribution of bremsstrahlung radiation. The values of the parameters c and d finely adjust the behavior of the attenuation curve of the LBRF.

If the value of parameter d is zero, that is, the three-parameter formula, the effect of rapid attenuation of the LBRF by x^{b+dx} at distances far from the source must transfer to parameters b and c. It follows that the value of parameter b does not represent the slope of the attenuation of the LBRF at a distance near the source, and the physical meaning of parameter b disappears.

4. Interpolation of the Fitted Response Function in the Distance, in the Emitted Angle (Φ), and in the Source Energy (E)

If there are some questionable values in the basic data, the statistical error for these are greater than the others in the fitting procedure. For such cases, the values of the parameters were determined for the basic data without any questionable data; questionable data can be replaced by the interpolated LBRFs calculated from these parameters.

An interpolation scheme proposed by Akima[28] was used to make the approximate LBRF continuous for both the source energy and the emitted angle. The interpolation scheme for the source energy was assured from a comparison between the LBRFs calculated by the interpolated four parameters of 2, 3, 5, and 7 MeV and those calculated by the EGS4 code at 4 MeV. They are in good agreement at all emitted angles of 4 MeV.

The present values of the four parameter were chosen so as to reflect the behaviors of the attenuation of the LBRF at a distance near the source and far from it, to adjust the maximum fractional deviation as small as possible, and to give smoothness regarding the emitted angle and the source energy.

The approximate LBRF is obtained by interpolation of the fit parameters.

5. Fitting to the CBRF Data for Neutron Source Energy and Secondary Gamma-Ray by the Four-Parameter Formula

There is the following relation between the emitted angle (Φ) and the polar angle (q):

$$\cos\Phi = \sin q \bullet \cos f, \qquad (11)$$

where q is measured from the vertical axis and f is the azimuthal angle measured from the source-detector axis.

The physical meanings of the parameters in the four-parameter formula are explained for a specified emitted angle (Φ). A fixed $\sin q$ corresponds to various Φ and f restricted by Eq.(11). That is, CBRF $R(E,q,x)$ is obtained by integrating the LBRF $R(E,\Phi,x)$ within the boundary satisfied by Eq.(11). Therefore, the behavior of the CBRF attenuation can not be treated with a clear method, such as that for the LBRF. However, the CBRF as a function of q varies more gently than the LBRF as a function of Φ. At first, the CBRFs data were fitted to an exponential formula,[17] $D(r) = Q / r \bullet \exp(-r / l)$, where Q and l are fitting parameters dependent on the energy and emission angle of the neutron beam, and r is the distance from the source in meters. Because this formula uses only two parameters, the approximated CBRF for a 400MeV neutron source causes overestimations at distances shorter than 100 m for neutrons and 200 m for gamma-rays. This distance tends to be longer when the incident neutron energy becomes higher. In the case of a 3-GeV neutron source, the limiting distance becomes 250 m for neutrons and 400 m for gamma-rays.[19)]

In order to accurately reproduce the values of the CBRF at a distance near the source, a trial should be made as to whether the CBRF approximated by the four-parameter formula accurately reproduces the reference CBRF.

Thus, the four-parameter formula was applied to approximate the CBRFs for neutrons and secondary gamma-rays. The results for neutrons of 3 and 1 GeV as well as 100, 10, 1, 0.1, 0.001 MeV and thermal are shown in **Figs. 5 and 6**, respectively, for neutron and secondary gamma-ray CBRFs. The variations of each fitting parameter for the CBRF as a function of the emitted cosine angle are less than those for the LBRF. The maximum deviations are within 17% for neutrons and within 12% for secondary gamma-rays for a neutron source energy above 1 MeV. However, the maximum fractional deviations are about 27% for secondary gamma-rays with the neutron source energy below 0.1 MeV. As a result, the calculated neutron and secondary gamma-ray dose distributions are better fitted to the four-parameter formula than the two-parameter formula, $D(r) = Q / r \bullet \exp(-r / l)$, including the distance near the source. The calculation of the CBRFs for neutrons and secondary gamma-rays can be accurately approximated by interpolation of the fitting parameter at an arbitrary distance and emitted cosine angle.

V. Conclusions

The calculations of the LBRFs for gamma-rays in skyshine analyses could be accurately approximated by the four-parameter empirical formula at an arbitrary distance, emitted angle, and energy. Physical meanings of the two parameters, a and b, in the four-parameter formula approximating the skyshine response function were established. That is, the parameter a always well reflected the values of LBRF at $x = 1$ meter, and the parameter b represented the attenuation ratio of LBRF at a distance near to the source. Also, the behavior of the attenuation at a distance near the source was explained by a single scattering photon.

The damping factor, $[\log f(x)]/x$, was represented by a linear equation as a function of $\log x$, where x is the distance of the source-to-detector in meters. The values of parameters c and d finely adjusted the behavior of the attenuation curves of the LBRF at a distance far from the source.

The calculated CBRFs for neutrons and secondary gamma-rays were better fit to the four-parameter empirical formula than the two-parameter empirical formula,[17] especially for distances near the source. Then, the CBRF could be accurately approximated by an interpolation of the fit parameters at an arbitrary distance and emitted cosine angle.

The values of the four parameters for gamma-ray LBRFs will be published soon by the High Energy Accelerator Research Organization and those for neutron and secondary gamma-ray CBRFs will be published by the Japan Atomic Energy Research Institute.

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Table 1: Values of the a parameter obtained from Eq. (5) and the b parameter obtained from Eq. (2) for a source energy of 1 MeV

Degree	a	b
1	-12.4035	-1.0665
15	-15.6211	-1.0704
30	-16.9130	-1.0437
60	-18.6216	-0.9651
90	-19.1355	-1.0518
135	-19.8242	-1.0057
180	-19.8870	-1.0476

Table 2 Parameters in the four-parameter formula for the gamma-ray line beam response function and comparison to values calculated by the EGS4 code

Source Energy 1.0 MeV						
Φ (deg)	LBRF Fit Parameters				Dev. Aver. (%)	Dev. Max (%)
	a	*b*	*c*	*d*		
0.0	- 9.2665	- 1.0168	- 6.864E-3*	- 0.098E-3	3.92	5.73
0.1	- 10.036	- 1.0396	- 7.106E-3	- 0.050E-3	3.62	5.62
1.0	- 12.354	- 1.0464	- 6.224E-3	- 0.120E-3	1.89	3.18
2.5	- 13.334	- 1.0411	- 5.317E-3	- 0.211E-3	1.05	1.71
5.0	- 14.130	- 1.0360	- 4.100E-3	- 0.351E-3	1.27	2.29
10	- 15.066	- 1.0294	- 2.659E-3	- 0.522E-3	1.21	2.01
15	- 15.669	- 1.0235	- 1.796E-3	- 0.637E-3	2.21	3.23
20	- 16.162	- 1.0200	- 0.919E-3	- 0.758E-3	1.74	2.88
30	- 16.975	- 1.0140	0.591E-3	- 0.984E-3	2.20	3.84
40	- 17.555	- 1.0091	1.200E-3	- 1.109E-3	2.99	4.34
50	- 18.039	- 1.0068	1.750E-3	- 1.233E-3	3.71	5.41
60	- 18.450	- 1.0018	2.200E-3	- 1.352E-3	3.94	5.81
70	- 18.785	- 1.0000	2.662E-3	- 1.472E-3	3.78	6.31
80	- 19.036	- 1.0000	3.096E-3	- 1.603E-3	3.80	6.52
90	- 19.236	- 1.0000	3.338E-3	- 1.701E-3	4.36	6.38
100	- 19.384	- 1.0000	3.413E-3	- 1.784E-3	3.49	5.90
120	- 19.615	- 1.0000	3.399E-3	- 1.882E-3	4.51	6.94
140	- 19.810	- 1.0000	3.408E-3	- 1.957E-3	3.31	4.76
160	- 19.897	- 1.0000	3.410E-3	- 2.017E-3	4.04	6.29
180	- 19.910	- 1.0000	3.407E-3	- 2.027E-3	4.45	6.24

*E-3 = 10^{-3}

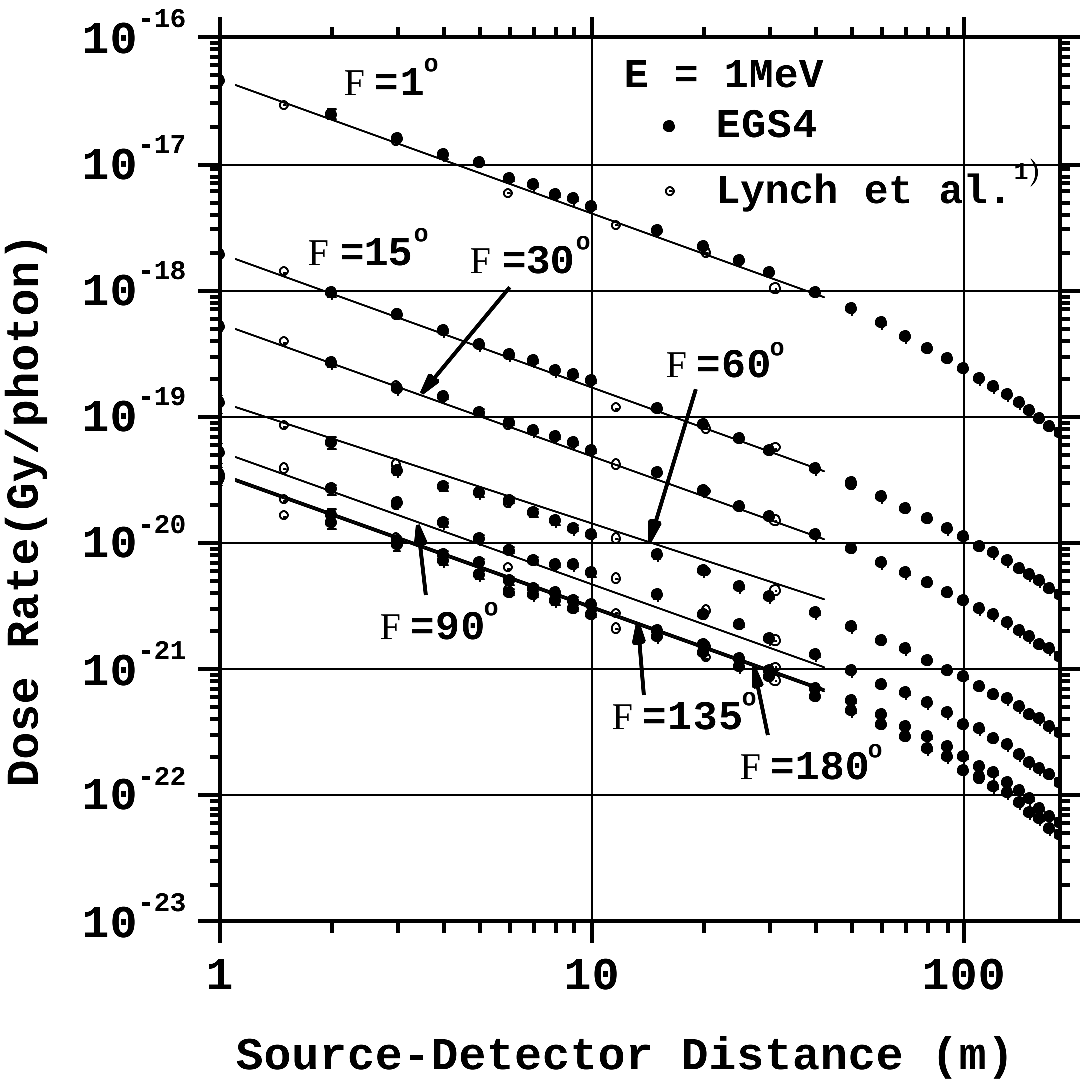

Fig. 1 LBRFs calculated with EGS4 as a function of the distance from a 1-MeV point mono-directional gamma-ray source together with the results of Lynch et al.[1)]

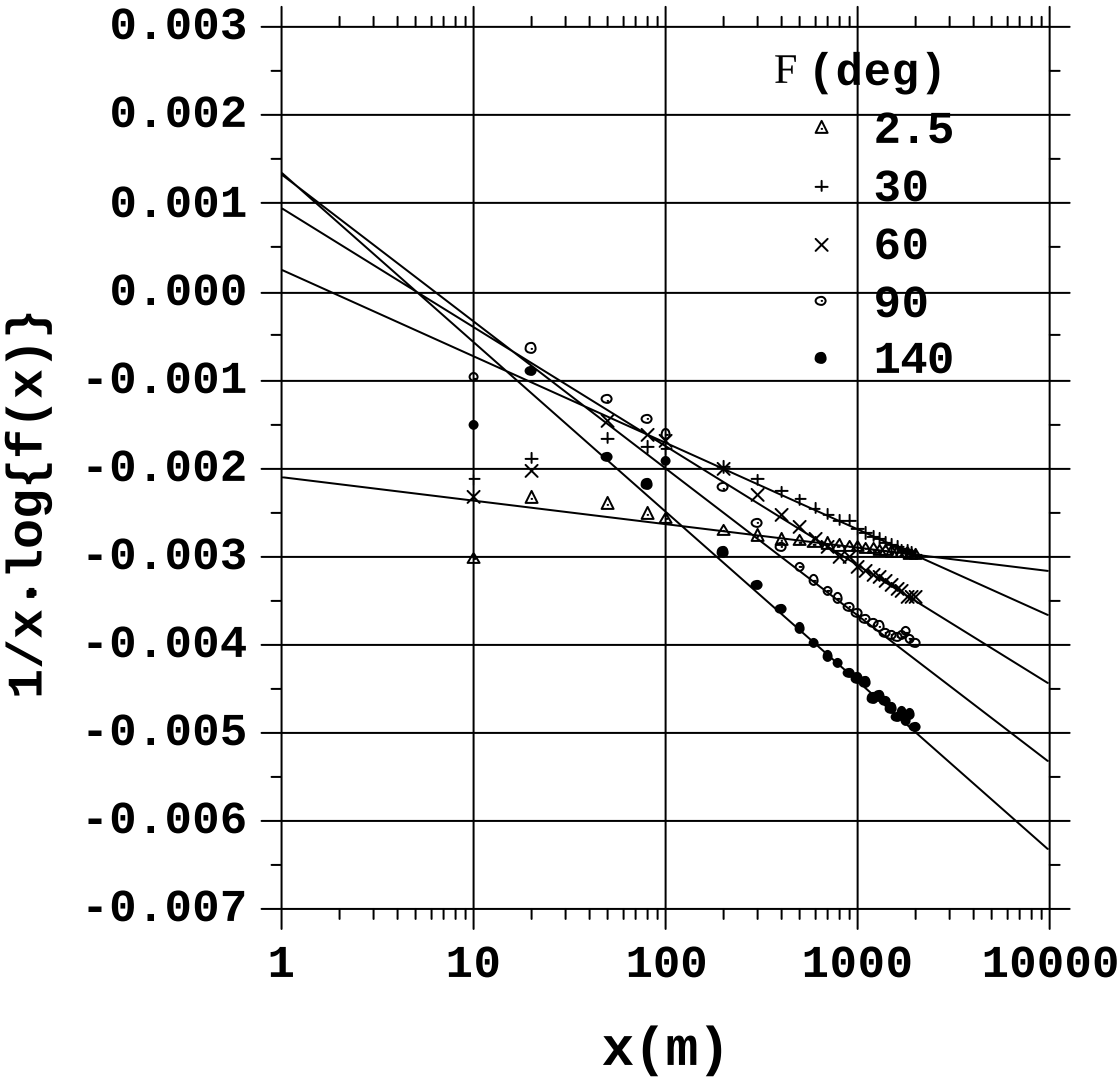

Fig. 2. $[\log f(x)]/x$ as a function of the distance for emitted angles of 2.5, 30, 60, 90 and 140 deg from a 1 MeV point mono-directional gamma-ray source .

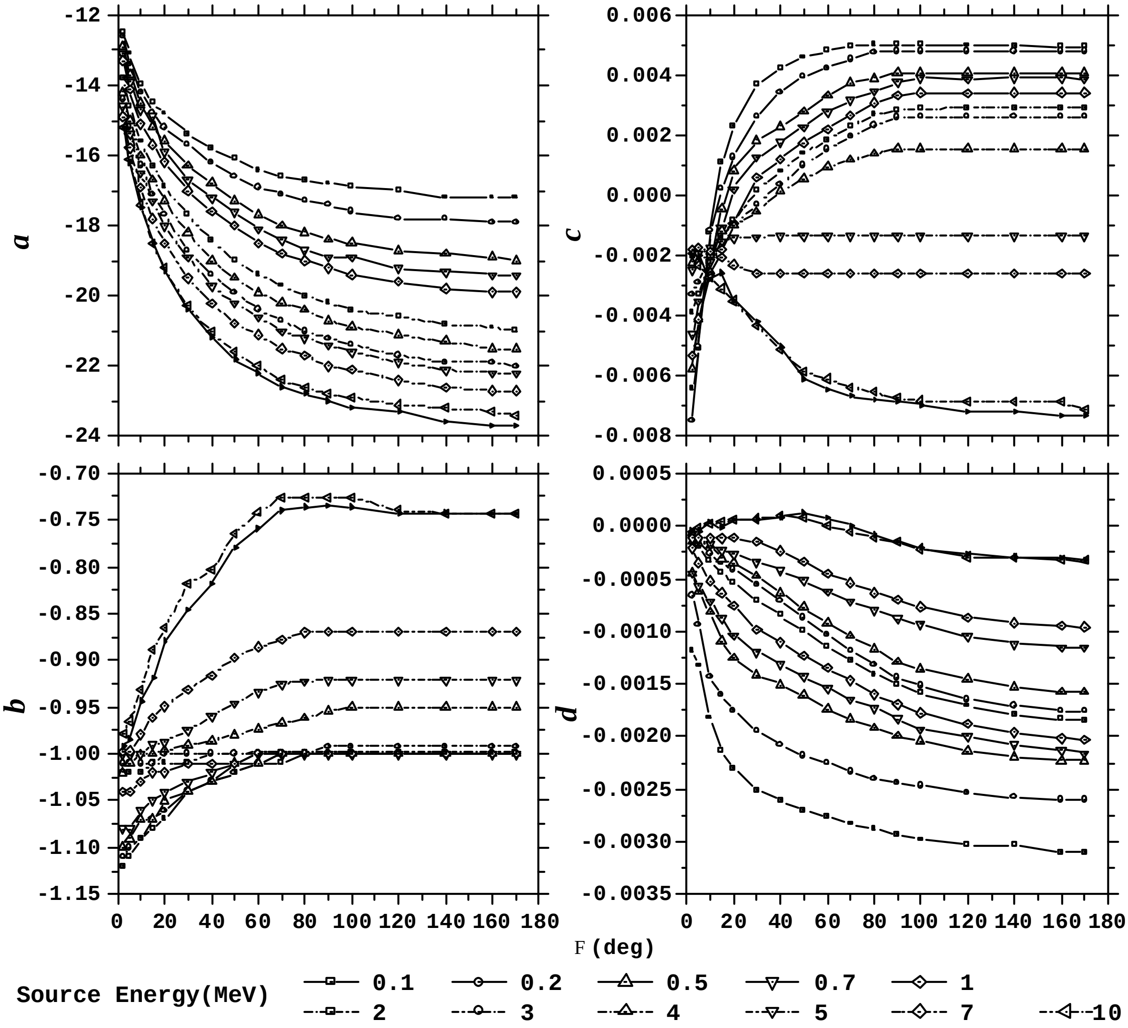

Fig. 3 Values of parameters a, b, c, and d in the four-parameter formula for the gamma ray LBRF as a function of the emitted angle for various source energies. for the fit parameter to the LBRF without bremsstrahlung radiation.

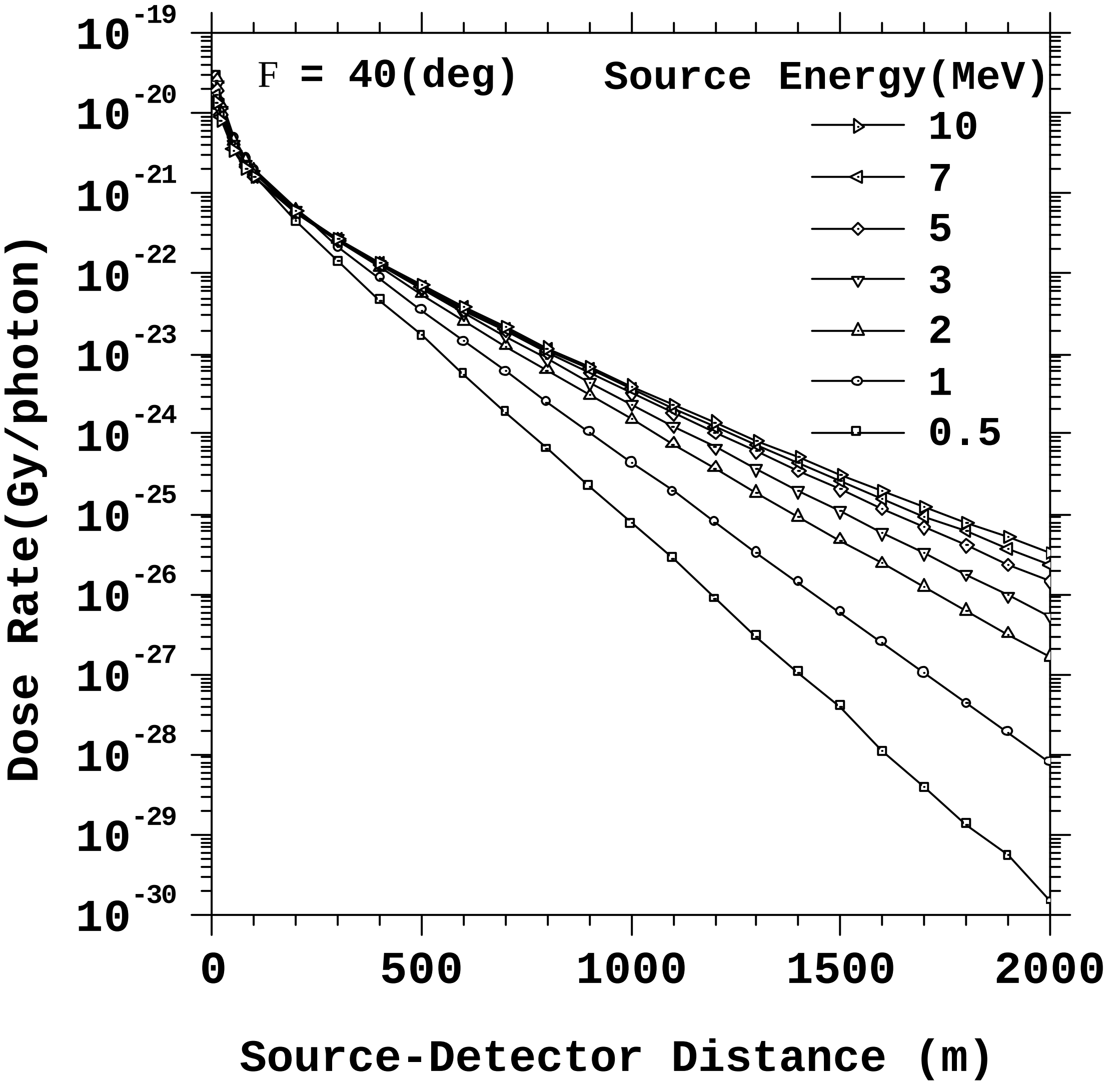

Fig. 4 Attenuations of the LBRFs at an emitted angle of 40 deg for various source energies

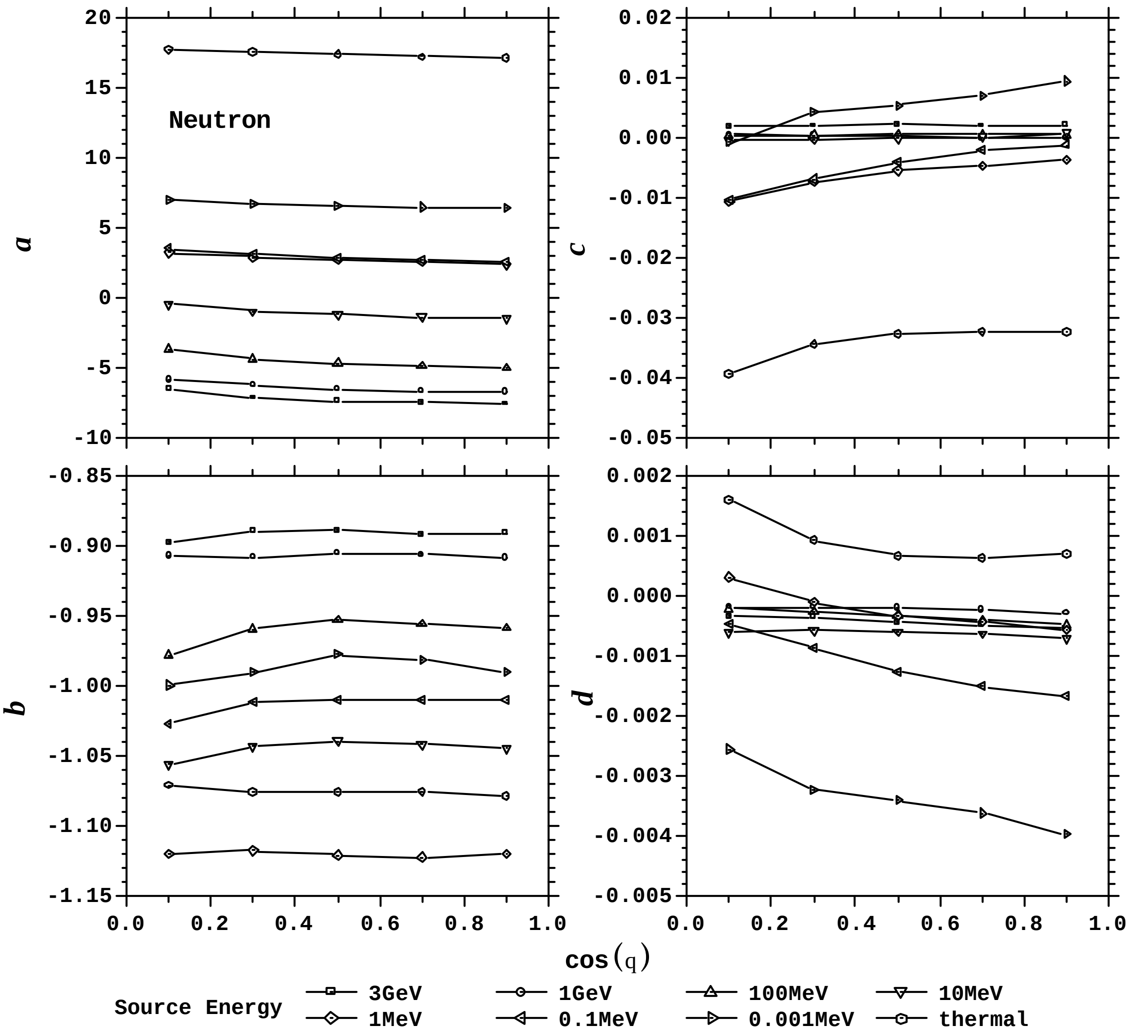

Fig. 5 Values of parameters a, b, c, and d in the four-parameter formula for the neutron CBRF as a function of emitted cosine angle for various source energies.

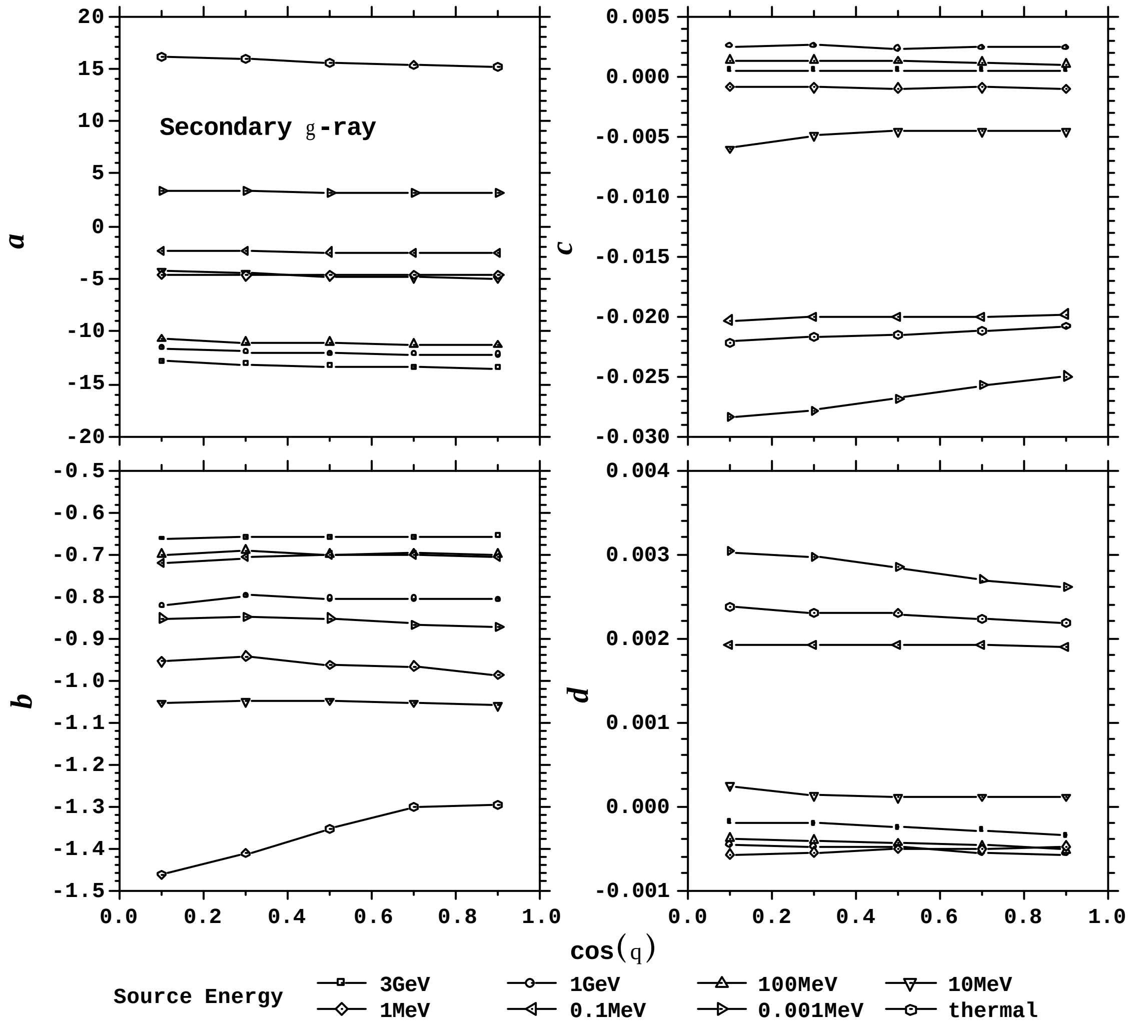

Fig. 6 Values of parameters a, b, c, and d in the four-parameter formula for the secondary gamma ray CBRF as a function of emitted cosine angle for various source energies.