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Lecture Notes of Radiation Transport Calculation Including Electrons by Monte Carlo Method

(English Version)

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English Parts

1 Monte Carlo Method

A method used to solve a problem with random numbers is called a "Monte Carlo Method".

1.1 Random numbers

Random numbers are a key tool for the Monte Carlo method. It is required to produce random numbers quickly when necessary. There are several ways to produce random numbers:

1. Use a dice, a roulette etc. — very slow.
2. Use a table of random numbers.
   - A table of random numbers has been well examined concerning its statistical characteristics.
   - It is required to store a whole table in computer data storage.
   - It currently is not very fast to produce random numbers.
3. Use physical random numbers like the decay of a radioisotope.
   - It is not easy to digitalize, and has a weakness concerning stability and reproducibility.
4. Produce random numbers successively from a seed random number, R_0, using a recurrence formula (a congruence equation in ordinary) in the form of $R_{n+1} = f(R_n)$. (pseudo-random numbers).
   - It is possible to produce the same random number sequences if the seed random number is the same.
   - Pseudo random numbers residuals by a divider, m.
   - There are m different integers at most and, therefore, pseudo random numbers have a limited period.
   - Good pseudo random numbers have the following features:
     (a) fast to create a random number
     (b) a long sequence
     (c) reproducibility
     (d) good statistical characteristics
   - It is possible to create pseudo random numbers between 0 and 1 by dividing pseudo random numbers by m.
5. There is another type of random-number generator called the Marasaglia-Zaman random-number generator[1]. It has a long periodicity ($2^{144} \sim 10^{43}$), and is portable to all 32-bit machines.

1.2 Pseudo random numbers

A linear congruence methods proposed by D. H. Lehmer is most widely used to produce pseudo random numbers:

$$R_{n+1} \equiv mod(aR_n + b, m) \qquad (n = 0, 1, ..., m),$$

where a, b and m are positive integers and a divider m is the length of the integer value allowed in the compiler ($m = 2^{31}$ is used for a 32 bit case).

Pseudo random numbers frequently used in Monte Carlo calculations and their a, b and m are given in Table 1.

Table 1. Names of pseudo random numbers and their a, b and m.

Name	a	b	m
RANDU	65539	0	2^{31}
SLAC RAN1	69069	0	2^{31}
SLAC RAN6	663608491	0	2^{31}

Another type of random number generators having a longer periodicity are used recently.

1. Marasaglia-Zaman random number generator[1]
   - Long periodicity – $2^{144} \sim 10^{43}$
   - Portable to all 32-bit machines
2. RANLUX random number generator[2]
   - Long periodicity – 10^{171}
   - Ranlux can produce independent random numbers by selecting a seed between 1 and 231.

1.3 Production of pseudo random numbers using a pocket calculator

1. Produce 10 random numbers for $R_0 = 3$, $a = 5$ and $m = 16$.
2. Confirm that the same sequence appears from some point.
   A number of random numbers produced until the same sequence appears is called a "sequence".
3. What is a sequence in this case ?
4. Check for a different R_0.

n	R_n	$R_n * 5$	$R_{n+1} = mod(R_0 * 5, 16)^*$
0	3		
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

$^*mod(R_0 * 5, 16) = R_0 * 5 - INT(\frac{R_0*5}{16}) * 16$

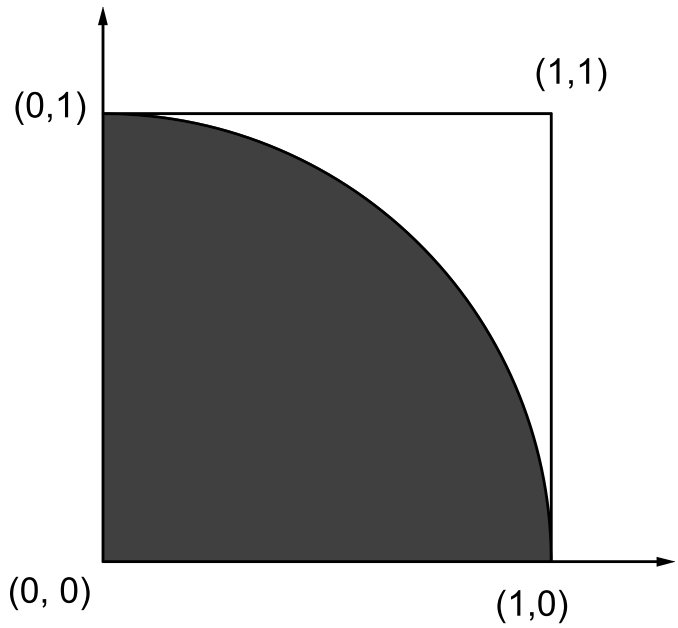
1.4 Calculation of π using random numbers

Select 2 random numbers between 0 and 1 in order starting from an arbitrary place in Table 2, which is created by SLAC RAN6, and count the number of pairs which satisfy the following condition.

$$R = \sqrt{\xi^2 + \eta^2} \leq 1.0$$

Trial number	ξ	η	R	$R \leq 1$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
				(A)
A/10=		(A/10)*4=		

A fraction ($A/10$) which satisfies the condition corresponds to the area within a circle of radius 1cm in a square of 1 cm. This is $\pi/4$ and, therefore, $\pi = 4 \times A/10$.



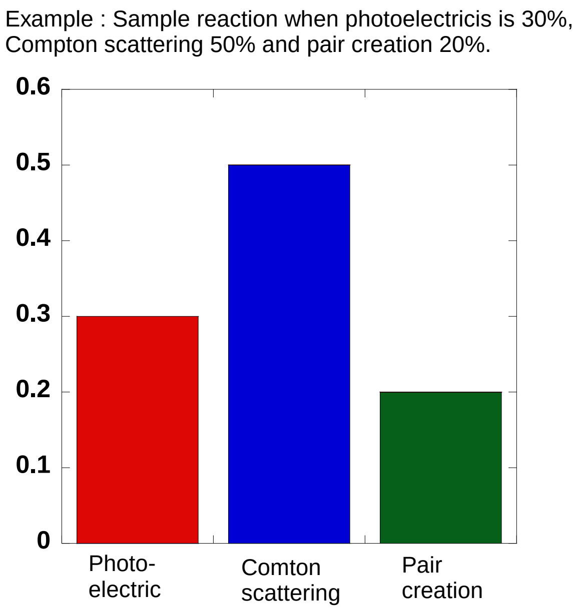
2 Radiation Transport by the Monte Carlo Method

Radiation trajectories are followed in a Mote Carlo calculation by determining each physical process with probability variables which describe each process.

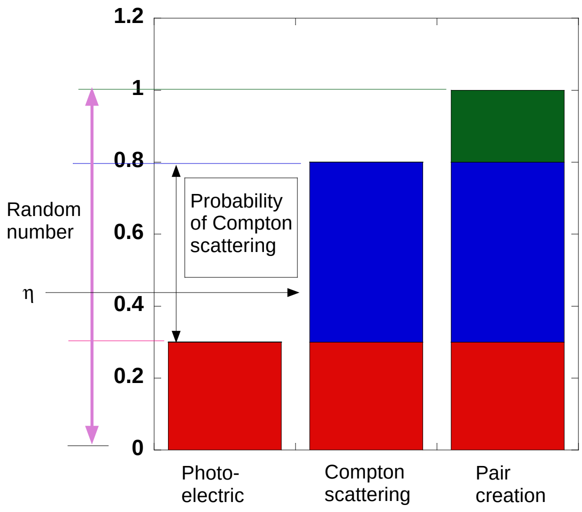
2.1 Sampling method

2.1.1 Discrete probability process

Example of a discrete probability process Procedures to sample a type of interaction when the probabilities of the photoelectric effect, Compton scattering and pair creation at a photon interaction are 30%(P_{photo}), 50%(P_{Compt}) and 20%(P_{pair}), respectively.



- The cumulative distribution function can be derived as follows from interaction probabilities.
- Photoelectric effect 0.3 (P_{photo})
- Compton scattering 0.3+0.5 ($P_{photo} + P_{Compt}$)=0.8
- Paire creation 0.3+0.5+0.1 ($P_{photo} + P_{Compt} + P_{pair}$)=1.0
- Random number variables is η.
- If $\eta \leq 0.3$, the reaction is a photoelectric.
- If $0.3 < \eta \leq 0.8$, the reaction is Compton scattering.
- If $0.8 < \eta$, the reaction is pair creation.



Cummulative probability function (pdf)

- $P_{photo} + P_{Compt} + P_{pair} = 1.0$
- If $\eta \leq P_{photo}$, the reaction is a photoelectric.
- If $P_{photo} < \eta \leq P_{photo} + P_{Compt}$, the reaction is Compton scattering.
- If $P_{photo} + P_{Compt} < \eta$, the reaction is pair creation.

General treatment of a discrete probability process If a probability variable (x) takes on discrete values (x_i) with probabilities (p_i) such that

$$F(x_n) = \sum_{i=1}^{n} p_i = 1,$$

$x = x_i$ if

$$F(x_i) = \sum_{j=1}^{i} p_j \leq \eta < F(x_{i+1}) = \sum_{j=1}^{i+1} p_j,$$

2.1.2 Continuous probability process

A probability distribution function (PDF:$f(x)$) for each physical process is defined over the range $[a, b]$, where neither a nor b is necessary finite. A PDF must have the properties such that it is both integrable and non-negative.

We now construct its cumulative probability function (CDF:$F(x)$),

$$F(x) = \int_a^x f(x_i)dx_i,$$

and assume that it is properly normalized, *i.e.* $F(b) = 1$.

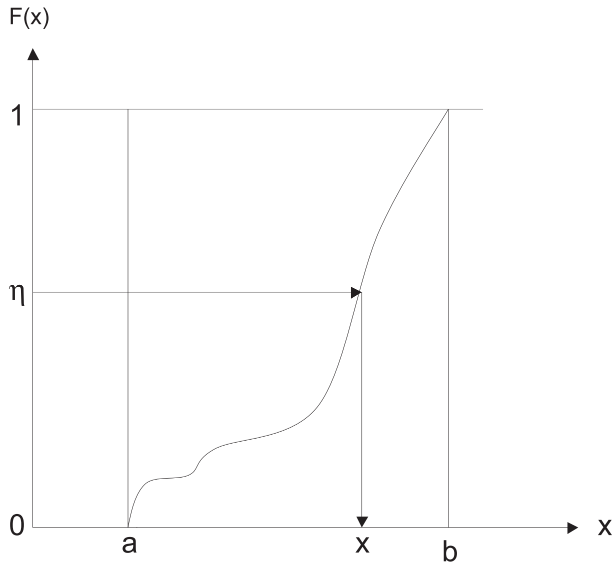
By its definition, we can map $F(x)$ onto a range of random variables, η, where $0 \leq \eta \leq 1$.

Having mapped the random numbers onto $F(x)$, we may invert the equation to give

$$x = F^{-1}(\eta).$$

The way to determine x by solving the above equation is called a "direct method".

In general, various techniques are necessary to determine x from the above equation.



Example of a direct method–determination of flight distance A particle interaction position is determined as follows:

1. If the interaction probability of a particle per unit distance is Σ_t, the number of decreases (dn) after dl is given by

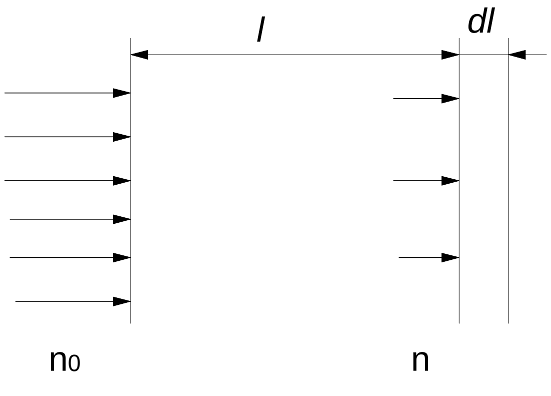
$$dn = -n\Sigma_t dl.$$

Therefore,

$$\int_{n_0}^{n} \frac{dn}{n} (= \ln \frac{n}{n_0}) = \int_0^l (-\Sigma_t) dl (= -\Sigma_t l),$$

$$\frac{n}{n_0} = e^{-\Sigma_t l},$$

where n_0 is the number of particles at $l = 0$.



2. $e^{-\Sigma_t l}$ is the probability that a particle does not interact within distance l. Therefore, the probability that a first interaction occurs between l and $l + dl$ is

$$p(l)dl = e^{-\Sigma_t l} \Sigma_t dl$$

and

$$\eta = P(l) = \int_0^l p(l_1) dl_1 = 1 - e^{-\Sigma_t l},$$

where η is a random number between 0 and 1.[1]

[1] $\int_0^\infty p(l)dl = 1$

3. By solving this equation, the flight distance (l) can be determined as

$$l = -\frac{1}{\Sigma_t} \ln(1 - \eta) = -\lambda \ln(1 - \eta).$$

$\lambda = 1/\Sigma_t$ is called as the "mean free path".

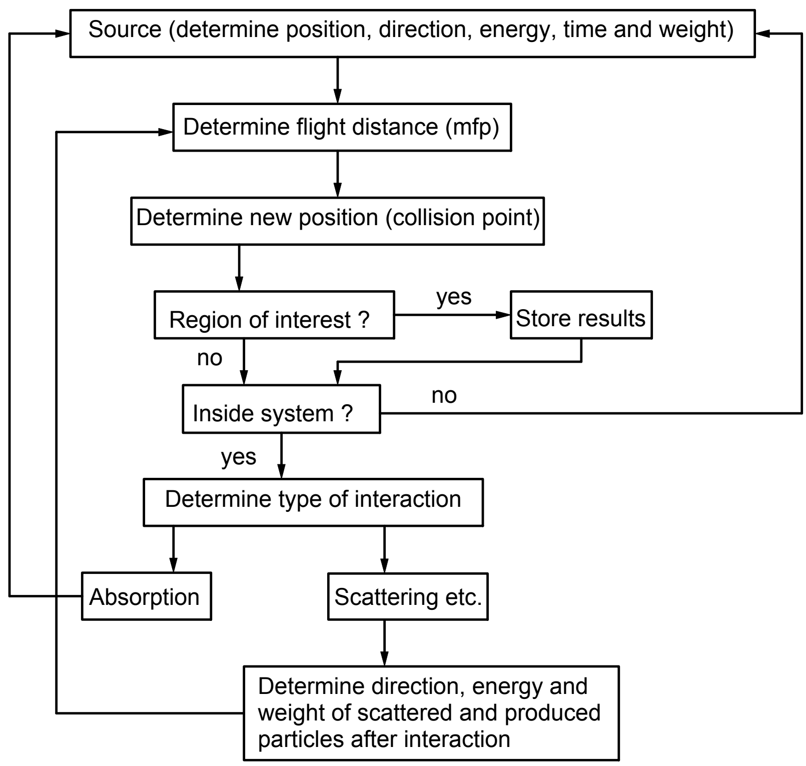
4. Considering that $1 - \eta$ is equivalent to η, l is usually determined by

$$l = -\lambda \ln \eta.$$

2.2 Simulation of radiation transport inside media

Source radiation simultaneously moves inside media while changing its position, direction and energy by scattering until it is absorbed. It is possible to obtain information like the number of particles or the absorbed energy at a specified region by the Monte Carlo method.

A basic flowchart of the Monte Carlo method is as follows:



1. Determine the source parameters.
    - position coordinates
    - direction coordinates
    - energy
    - weight
2. Determine the distance to a interaction point, the flight distance (l), using the total cross section.
3. Check whether an interaction point is within the same region or not.
    - Uncharged particle, like photons or neutrons, move to an interaction point without changing its direction or energy. In this case, this is a comparison between the flight distance (l) and the distance to the region boundary (d).
        (a) If $l < d$, move the particle to the interaction point.
        (b) If $l \geq d$, move the particle to the boundary.
            – If the medium of the new region is the same, set the flight distance to $l - d$ and repeat the same procedure. Otherwise, determine the flight path for the new medium.
            – If the new region is outside the system of interest, stop following this particle and produce a new particle.

   - A charged particle, like an electron, changes its direction and energy while moving to the interaction point and, therefore, treatments become more complicate.

4. Determine the type of interaction.
   - The type of interaction is determined using discrete-type probability distribution functions.
   - Photoelectric or Compton scattering or pair production is selected in the case of photons.
5. Determine the energy and a direction of scattered and produced particles at the interaction point using the differential cross section of the interaction.
6. Store any information of interest when a particle reaches to region of interest, such as:
   - type of particle and its energy,
   - energy imparted to the medium.
7. Terminate following radiation when
   - radiation leaks from the system or
   - the radiation energy becomes below its cut-off energy.
8. A history is defined as the whole processes from the production of a source particle until its termination for some reason. Information of interests can be obtained by repeating a history many times.

3 A Simple Example of Radiation Transport

3.1 Single layer

Consider uniform medium, A, of 50 cm thickness (see Fig. 1).

1. Suppose that
   - 0.5MeV photons enter on this system from the left end,
   - the mean free path is 20 cm,
   - the ratio of the photoelectric effect and Compton scattering is 1:1, and
   - a scattered photon does not change its energy or direction.
2. Example 1
   - Suppose that a first random number is .234.
   - $l = -20 \times \ln 0.234 = 29.0$ (1mfp=20 cm)
   - $29.0(cm) < 50.0(cm)$
   - Next random number is 0.208 and less than 0.5. $-- >$ Photo-electric effect
3. Example 2
   - Next random number is 906.
   - $l = -20 \times \ln 0.906 = 1.97$ (1mfp=20cm)
   - $1.97(cm) < 50.0(cm)$
   - Next random number is 0.716 and larger than 0.5.$-- >$ Compton scatterimg

- Next random number is 0.996.
- $l = -20 \times \ln 0.996 = 0.0802$ (1mfp=20cm)
- $0.0802(cm) < 50.0 - 1.97 = 48.03(cm)$
- Next random number is 0.600 and larger than 0.5.−− > Compton scatterimg
- Next random number is 0.183.
- $l = -20 \times \ln 0.183 = 34.0$ (1mfp=20cm)
- $34.0(cm) < 48.03 - 0.0802 = 47.95(cm)$
- Next random number is 0.868 and larger than 0.5.−− > Compton scatterimg
- Next random number is 0.351.
- $l = -20 \times \ln 0.351 = 20.9$ (1mfp=20cm)
- $20.9(cm) > 47.95 - 34.0 = 13.95(cm)$
- Terminate due to escape from the system.

4. Starting from an arbitrary random number in Table 2, follow 10 photons like an example in Table 3, and count the number of photons transmitted in a plane.

5. Make trajectories of particles like an example in Fig. 1.

3.2 Double layer

Consider 30 cm of medium A followed by 20 cm of medium B (see Fig. 2).

1. Suppose that:

   - 0.5MeV photons enter this system from the left end,
   - the mean free path and the ratio of the photoelectric effect and Compton scattering in medium A are same as in the previous case,
   - the mean free path of medium B is 3cm,
   - the ratio of the photoelectric effect and Compton scattering of medium B is 3:1, and
   - a scattered photon does not change its energy or direction for both media.

2. Example 1

   - First random number is 0.329.
   - $l = -20 \times \ln 0.329 = 22.2$ (1mfp=20cm)
   - $22.2(cm) < 30.0(cm)$
   - Next random number is 0.612 and larger than 0.5.−− > Compton scatterimg
   - Next random number is 0.234.
   - $l = -20 \times \ln 0.234 = 29.0$ (1mfp=20cm)
   - $29.0(cm) > 30.0 - 22.2 = 7.8(cm)$
   - Move to boundary (30.0cm).
   - Next random number is 0.281.
   - $l = -3 \times \ln 0.281 = 3.81$ (1mfp=3cm)
   - $3.81(cm) < 20.0(cm)$
   - Next random number is 0.906 and larger than 0.75.−− > Compton scatterimg
   - Next random number is 0.716.

- $l = -3 \times \ln 0.716 = 1.00$ (1mfp)=3cm)
- $1.00(cm) < 20.0 - 3.81 = 16.19(cm)$
- Next random number is 0.996 and larger than 0.75.$-- >$ Compton scatterimg
- Next random number is 0.600.
- $l = -3 \times \ln 0.600 = 1.53$ (1mfp=3cm)
- $1.53(cm) < 16.19 - 1.00 = 15.19(cm)$
- Next random number is 0.183 and less than 0.75.$-- >$ Photoelectric Effecct.
- Teminate history.

3. Starting from an arbitrary random number in Table 2, follow 10 photons, like the example in Table 4, and count the number of photons transmitted in medium B.

4. Make trajectories of particles, like the example given in Fig. 2.

4 Complex, but More Realistic, Example of Radiation Transport

Consider the 10 cm aluminum plane shown in Fig. 3.

Suppose that

1. 0.5MeV photons enter this system from the left end,
2. Photons are scattered with equal probability for each 90° at Compton scattering for all photon energies,

Scattering Angle	Probability
0°	100/3 %
90°	100/3 %
180°	100/3 %

3. The photon energy after scattering is calculated by

$$E = \frac{E_0}{1 + \left(\frac{E_0}{0.511}\right)(1 - \cos\theta)},$$

where E_0(MeV) is the photon energy before scattering, E(MeV) is that after scattering and θ is the scattering angle.

4. Suppose that the azimuthal angle after Compton scattering is 0° or 180° with an equal probability. 0° is 90° left from the particle direction and 180° is 90° right.
5. Use the mean free path (mfp) and branching ratio for each photon energy in Figs. 4 and 5.
6. Set the cutoff energy of photons to 0.05MeV.

4.1 Example

The example in Table 5 can be explained as follows:

- Source photon
  1. The mfp of 0.5MeV is 4.15cm from Fig. 4.
  2. If we start a random number from 0.351 in Table 3, the flight distance of this photon is
  $$l = -\ln(0.351) * 4.15 = 4.33(cm).$$
  3. This distance is smaller than that to the boundary (10cm). The reaction point is therefore inside the Al plane.
  4. The probability of a photoelectric reaction for 0.5MeV is 0.0018 from Fig. 5.
  5. The next random number is 0.259, which is larger than 0.0018. Therefore, the reaction is Compton scattering.
  6. Next, determine the scattering angle. The scattering angle is 0° if a random number is smaller than 1/3, 90° if it is between 1/3 and 2/3 and 180° if it is larger than 2/3. The next random number is 0.572. Therefore, the scattering angle is 90°.

7. Calculate photon energy after scattering.

$$E = \frac{0.5}{1 + \left(\frac{0.5}{0.511}\right)(1 - \cos 90^\circ)} = 0.252(MeV)$$

8. The azimuthal angle is 0° if the random number is less than 0.5, and is 180° otherwise. The next random number is 0.888, and therefore the azimuthal angle is 180°.

- Scattered photon after the first interaction

  1. The mfp of 0.25MeV is 3.34cm from Fig. 4.
  2. The next random number is 0.238 and the flight distance is

  $$l = -\ln(0.238) * 3.34 = 4.79(cm).$$

  3. The plane is infinite for the X-direction. Therefore, an interaction occurs within the Al plane.
  4. The probability of a photoelectric reaction for 0.25MeV is 0.01 from Fig. 5.
  5. The next random number is 0.669, which is larger than 0.01. Therefore, the reaction is Compton scattering.
  6. The next random number is 0.0478 and the scattering angle is 0°. For $\theta = 0^\circ$, a photon does not change energy and it is not necessary to determine the azimuthal angle.
  7. The photon moves from the position of X=-4.79 cm and Z=4.34 cm to the direction of -X.

- Scattered photon after a second interaction

  1. The mfp of 0.25MeV is 3.34cm, the same as in the previous case.
  2. The next random number is 0.949 and the flight distance is

  $$l = -\ln(0.949) * 3.34 = 0.175(cm).$$

  3. The plane is infinite for the X-direction. Therefore, an interaction occurs within the Al plane.
  4. The probability of a photoelectric reaction for 0.25MeV is 0.01, the same as in the previous case.
  5. The next random number is 0.324, which is larger than 0.01. Therefore, the reaction is Compton scattering.
  6. The next random number is 0.579 and the scattering angle is 90°.
  7. Calculate the photon energy after scattering,.

  $$E = \frac{0.25}{1 + \left(\frac{0.25}{0.511}\right)(1 - \cos 90^\circ)} = 0.168(MeV).$$

  8. The next random number is 0.439, which is smaller than 0.5. Therefore, the azimuthal angle is 0°.
  9. The photon moves from the positions of X=-4.96 cm and Z=4.34 cm to the direction of Z.

4.2 Practices

1. Following the same procedure as shown above until a photoelectric effect occurs, the photon energy becomes below a cut-off energy or the photo reaches the boundary (Z $<$ 0.0 or Z $>$ 10 cm).

2. Start from another source photon and follow its movements as in the above example. Make trajectories of the photon in Fig. 6, like the example in Fig. 3.

3. Change the medium from Al to Fe. Start from a source photon and follow its movements, as in the above example. Make trajectories of the photon in Fig. 6 like the example in Fig. 3.

5 Electron Trajectories without Hard Collision

As mentioned before, an electron moves inside materials changing its direction via many elastic collisions with nucleus. It is almost impossible to treat each elastic collision seplately. Therefore, an electron trajectory is divided into many small steps and a curved moving a distance, direction and displacement after step are calculated based on the multiple scattering model. An electron also loses its kinetic energy via many inelastic collisions with atomic electrons and as the results ionizes or excites atoms.

Consider the 1 mm aluminum plane shown in Fig. 7.

Suppose that

1. A 1.0 MeV electron enters this system from the left end,
2. Neglect a correction of moving distance due to multiple scattering.
3. Hard collisions such as δ-ray or bremsstrahlung productions do not occur.
4. Set a step size of electron to 0.01 cm for all energy of electrons.
5. The electron changes its direction via multiple scattering with equal probability for each 90° for all electron energies.

Scattering Angle	Probability
0°	100/3 %
90°	100/3 %
180°	100/3 %

6. Suppose that the azimuthal angle after multiple scattering is 0° or 180° with an equal probability. 0° is 90° left from the particle direction and 180° is 90° right.
7. Use same energy loss of 0.04 MeV per 0.01 cm for all electron energies. (Stopping power of aluminum for 1 MeV electrons is 1.465(MeV cm^2/g). The energy loss at 0.01 cm is $0.01 \times 1.465 \times 2.7 = 0.0396$ MeV. Therefore, this assumption means that we use the electron stopping power at 1 MeV for all electron energies.)
8. Set the cutoff energy of electron to 0.01MeV.

5.1 Example

The example in Table 6 can be explained as follows:

1. 1 MeV electron move to Z=0.01 cm on the Z-axis.
2. This position (Z=0.01, X=0.0cm) is inside aluminum plane due to $0.0 \geq Z \leq 0.1$.
3. Calculate electron energy (E_1=0.96 MeV) after traveling 0.01cm by $1.0 - 0.04 = 0.96 MeV$.
4. Next, determine the scattering angle. The scattering angle is 0° if a random number is smaller than 1/3, 90° if it is between 1/3 and 2/3 and 180° if it is larger than 2/3. The first random number is 0.126. Therefore, the scattering angle is 0°.
5. The azimuthal angle is 0° if the random number is less than 0.5, and is 180° otherwise. The next random number is 0.938, and therefore the azimuthal angle is 180°.
6. The electron moves 0.01cm from the position of X=0.0 cm and Z=0.01 cm to the direction of Z.

7. This position (Z=0.02, X=0.0cm) is inside aluminum plane due to $0.0 \geq Z \leq 0.1$.

8. Calculate electron energy (E_2=0.92 MeV) after traveling 0.01cm by $0.96 - 0.04 = 0.92 MeV$.

9. The next random number is 0.643. Therefore, the scattering angle is 90°.

10. The next random number is 0.081, and therefore the azimuthal angle is 0°.

11. The electron moves 0.01cm from the position of X=0.0 cm and Z=0.02 cm to the direction of X.

12. This position (Z=0.02, X=0.01cm) is inside aluminum plane due to $0.0 \geq Z \leq 0.1$.

13. Calculate electron energy (E_3=0.88 MeV) after traveling 0.01cm by $0.92 - 0.04 = 0.88 MeV$.

14. The next random number is 0.556. Therefore, the scattering angle is 90°.

15. The next random number is 0.817, and therefore the azimuthal angle is 180°.

16. Follows electron trajectories by the same way. At n=25, electron energy becomes a below cut-off energy. Electron stop at this position.

5.2 Practice 1

1. Start from another source electron and follow its movements as in the above example until the electron energy becomes below a cut-off energy or the electron reaches the boundary ($Z < 0.0$ or $Z > 10$ cm).

2. Write trajectory information on Table 7 and make trajectories of the electron using Table 7 in Fig. 8, like the example in Fig. 7.

5.3 Practice 2

1. Start from another source electron and follow its movements as in the above example until the electron energy becomes below a cut-off energy or the electron reaches the boundary ($Z < 0.0$ or $Z > 10$ cm).

2. Use energy dependent electron stopping power to calculate energy deposition due to movement inside the aluminum.

   - Mass collision stopping power for aluminum are shown in Fig.9[3] as a function of electron energy. The value of mass collision stopping power (MeV cm^2/g) can be approximated by a following equation in the energy range from 0.01 to 1 MeV.

   $$\frac{1}{\rho}\frac{dE}{dx} = exp\left\{0.292 + 0.0293 * ln(E) + 0.194 * (ln(E))^2 + 0.0161 * (ln(E))^3\right\}$$

   - Collision stopping power can be calculated by multiplied the density of aluminum (ρ=2.7g/cm^3) to mass collision stopping power.

3. Write trajectory information on Table 8 and make trajectories of the electron using Table 8 in Fig. 10, like the example in Fig. 7.

References

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Table 2.a Pseudo random number between 0–1 (RAN6).

□□□ 0.896	□□□ 0.898	□□□ 0.392	□□□ 0.405	□□□ 0.784
□□□ 0.117	□□□ 0.710	□□□ 0.732	□□□ 0.565	□□□ 0.892
□□□ 0.105	□□□ 0.458	□□□ 0.670	□□□ 0.254	□□□ 0.284
□□□ 0.991	□□□ 0.909	□□□ 0.320	□□□ 0.126	□□□ 0.983
□□□ 0.642	□□□ 0.081	□□□ 0.556	□□□ 0.817	□□□ 0.501
□□□ 0.920	□□□ 0.896	□□□ 0.618	□□□ 0.759	□□□ 0.690
□□□ 0.251	□□□ 0.094	□□□ 0.371	□□□ 0.148	□□□ 0.492
□□□ 0.519	□□□ 0.789	□□□ 0.567	□□□ 0.397	□□□ 0.179
□□□ 0.576	□□□ 0.341	□□□ 0.517	□□□ 0.583	□□□ 0.909
□□□ 0.380	□□□ 0.326	□□□ 0.756	□□□ 0.021	□□□ 0.132
□□□ 0.224	□□□ 0.929	□□□ 0.646	□□□ 0.019	□□□ 0.937
□□□ 0.935	□□□ 0.530	□□□ 0.117	□□□ 0.906	□□□ 0.622
□□□ 0.074	□□□ 0.886	□□□ 0.199	□□□ 0.603	□□□ 0.164
□□□ 0.763	□□□ 0.526	□□□ 0.649	□□□ 0.260	□□□ 0.431
□□□ 0.914	□□□ 0.031	□□□ 0.795	□□□ 0.577	□□□ 0.600
□□□ 0.148	□□□ 0.959	□□□ 0.946	□□□ 0.719	□□□ 0.719
□□□ 0.922	□□□ 0.518	□□□ 0.329	□□□ 0.883	□□□ 0.558
□□□ 0.599	□□□ 0.351	□□□ 0.499	□□□ 0.744	□□□ 0.661
□□□ 0.983	□□□ 0.970	□□□ 0.275	□□□ 0.725	□□□ 0.147
□□□ 0.892	□□□ 0.482	□□□ 0.113	□□□ 0.534	□□□ 0.855
□□□ 0.598	□□□ 0.368	□□□ 0.807	□□□ 0.701	□□□ 0.944
□□□ 0.173	□□□ 0.536	□□□ 0.541	□□□ 0.987	□□□ 0.064
□□□ 0.402	□□□ 0.869	□□□ 0.350	□□□ 0.752	□□□ 0.264
□□□ 0.061	□□□ 0.814	□□□ 0.885	□□□ 0.627	□□□ 0.580
□□□ 0.400	□□□ 0.031	□□□ 0.088	□□□ 0.208	□□□ 0.563
□□□ 0.727	□□□ 0.314	□□□ 0.606	□□□ 0.595	□□□ 0.379
□□□ 0.116	□□□ 0.512	□□□ 0.271	□□□ 0.848	□□□ 0.188
□□□ 0.913	□□□ 0.810	□□□ 0.515	□□□ 0.067	□□□ 0.464
□□□ 0.225	□□□ 0.657	□□□ 0.874	□□□ 0.511	□□□ 0.107
□□□ 0.924	□□□ 0.410	□□□ 0.993	□□□ 0.910	□□□ 0.755
□□□ 0.155	□□□ 0.557	□□□ 0.813	□□□ 0.520	□□□ 0.746
□□□ 0.195	□□□ 0.199	□□□ 0.221	□□□ 0.482	□□□ 0.705
□□□ 0.837	□□□ 0.557	□□□ 0.438	□□□ 0.306	□□□ 0.804
□□□ 0.155	□□□ 0.728	□□□ 0.705	□□□ 0.240	□□□ 0.801
□□□ 0.497	□□□ 0.932	□□□ 0.966	□□□ 0.463	□□□ 0.199
□□□ 0.260	□□□ 0.056	□□□ 0.935	□□□ 0.714	□□□ 0.522
□□□ 0.404	□□□ 0.899	□□□ 0.890	□□□ 0.126	□□□ 0.363
□□□ 0.230	□□□ 0.041	□□□ 0.100	□□□ 0.509	□□□ 0.352
□□□ 0.995	□□□ 0.461	□□□ 0.601	□□□ 0.454	□□□ 0.226
□□□ 0.234	□□□ 0.790	□□□ 0.387	□□□ 0.661	□□□ 0.427

Table 2.b Pseudo random number between 0–1 (RAN6).

□□□ 0.145	□□□ 0.040	□□□ 0.695	□□□ 0.270	□□□ 0.566
□□□ 0.032	□□□ 0.001	□□□ 0.045	□□□ 0.125	□□□ 0.498
□□□ 0.685	□□□ 0.803	□□□ 0.919	□□□ 0.819	□□□ 0.347
□□□ 0.293	□□□ 0.492	□□□ 0.079	□□□ 0.624	□□□ 0.406
□□□ 0.879	□□□ 0.074	□□□ 0.759	□□□ 0.458	□□□ 0.346
□□□ 0.689	□□□ 0.771	□□□ 0.609	□□□ 0.879	□□□ 0.450
□□□ 0.787	□□□ 0.742	□□□ 0.499	□□□ 0.056	□□□ 0.091
□□□ 0.118	□□□ 0.444	□□□ 0.724	□□□ 0.470	□□□ 0.105
□□□ 0.301	□□□ 0.139	□□□ 0.392	□□□ 0.302	□□□ 0.138
□□□ 0.831	□□□ 0.605	□□□ 0.375	□□□ 0.705	□□□ 0.795
□□□ 0.706	□□□ 0.910	□□□ 0.760	□□□ 0.155	□□□ 0.245
□□□ 0.595	□□□ 0.591	□□□ 0.695	□□□ 0.925	□□□ 0.052
□□□ 0.789	□□□ 0.067	□□□ 0.463	□□□ 0.625	□□□ 0.337
□□□ 0.483	□□□ 0.678	□□□ 0.429	□□□ 0.080	□□□ 0.714
□□□ 0.356	□□□ 0.995	□□□ 0.636	□□□ 0.195	□□□ 0.470
□□□ 0.800	□□□ 0.808	□□□ 0.062	□□□ 0.305	□□□ 0.005
□□□ 0.567	□□□ 0.920	□□□ 0.061	□□□ 0.718	□□□ 0.663
□□□ 0.176	□□□ 0.484	□□□ 0.079	□□□ 0.920	□□□ 0.716
□□□ 0.101	□□□ 0.502	□□□ 0.297	□□□ 0.771	□□□ 0.613
□□□ 0.363	□□□ 0.757	□□□ 0.770	□□□ 0.010	□□□ 0.465
□□□ 0.018	□□□ 0.990	□□□ 0.971	□□□ 0.579	□□□ 0.244
□□□ 0.819	□□□ 0.114	□□□ 0.388	□□□ 0.738	□□□ 0.451
□□□ 0.651	□□□ 0.127	□□□ 0.710	□□□ 0.809	□□□ 0.025
□□□ 0.251	□□□ 0.163	□□□ 0.531	□□□ 0.069	□□□ 0.433
□□□ 0.976	□□□ 0.808	□□□ 0.277	□□□ 0.206	□□□ 0.242
□□□ 0.908	□□□ 0.721	□□□ 0.557	□□□ 0.920	□□□ 0.177
□□□ 0.205	□□□ 0.803	□□□ 0.865	□□□ 0.350	□□□ 0.191
□□□ 0.037	□□□ 0.300	□□□ 0.974	□□□ 0.082	□□□ 0.472
□□□ 0.800	□□□ 0.751	□□□ 0.409	□□□ 0.996	□□□ 0.824
□□□ 0.627	□□□ 0.497	□□□ 0.242	□□□ 0.897	□□□ 0.424
□□□ 0.159	□□□ 0.492	□□□ 0.468	□□□ 0.843	□□□ 0.992
□□□ 0.724	□□□ 0.529	□□□ 0.637	□□□ 0.835	□□□ 0.119
□□□ 0.049	□□□ 0.775	□□□ 0.944	□□□ 0.334	□□□ 0.287
□□□ 0.406	□□□ 0.454	□□□ 0.859	□□□ 0.045	□□□ 0.434
□□□ 0.695	□□□ 0.769	□□□ 0.927	□□□ 0.527	□□□ 0.907
□□□ 0.182	□□□ 0.664	□□□ 0.353	□□□ 0.938	□□□ 0.737
□□□ 0.565	□□□ 0.874	□□□ 0.471	□□□ 0.403	□□□ 0.165
□□□ 0.502	□□□ 0.349	□□□ 0.932	□□□ 0.122	□□□ 0.930
□□□ 0.565	□□□ 0.275	□□□ 0.429	□□□ 0.452	□□□ 0.469
□□□ 0.258	□□□ 0.017	□□□ 0.582	□□□ 0.761	□□□ 0.847

Table 2.c Pseudo random number between 0–1 (RAN6).

□□□ 0.779	□□□ 0.789	□□□ 0.837	□□□ 0.308	□□□ 0.117
□□□ 0.157	□□□ 0.037	□□□ 0.566	□□□ 0.047	□□□ 0.779
□□□ 0.373	□□□ 0.094	□□□ 0.930	□□□ 0.711	□□□ 0.288
□□□ 0.624	□□□ 0.901	□□□ 0.279	□□□ 0.012	□□□ 0.014
□□□ 0.048	□□□ 0.570	□□□ 0.083	□□□ 0.561	□□□ 0.410
□□□ 0.904	□□□ 0.585	□□□ 0.089	□□□ 0.847	□□□ 0.116
□□□ 0.674	□□□ 0.119	□□□ 0.865	□□□ 0.440	□□□ 0.953
□□□ 0.433	□□□ 0.428	□□□ 0.830	□□□ 0.252	□□□ 0.342
□□□ 0.852	□□□ 0.509	□□□ 0.388	□□□ 0.982	□□□ 0.815
□□□ 0.579	□□□ 0.454	□□□ 0.928	□□□ 0.570	□□□ 0.482
□□□ 0.208	□□□ 0.469	□□□ 0.399	□□□ 0.152	□□□ 0.124
□□□ 0.828	□□□ 0.400	□□□ 0.642	□□□ 0.661	□□□ 0.654
□□□ 0.634	□□□ 0.056	□□□ 0.320	□□□ 0.102	□□□ 0.730
□□□ 0.600	□□□ 0.052	□□□ 0.797	□□□ 0.982	□□□ 0.549
□□□ 0.568	□□□ 0.017	□□□ 0.021	□□□ 0.960	□□□ 0.131
□□□ 0.385	□□□ 0.109	□□□ 0.932	□□□ 0.376	□□□ 0.400
□□□ 0.129	□□□ 0.230	□□□ 0.727	□□□ 0.109	□□□ 0.328
□□□ 0.086	□□□ 0.986	□□□ 0.239	□□□ 0.874	□□□ 0.988
□□□ 0.625	□□□ 0.093	□□□ 0.297	□□□ 0.265	□□□ 0.385
□□□ 0.536	□□□ 0.863	□□□ 0.295	□□□ 0.704	□□□ 0.368

Table 3 Single layer.

No.	d(cm)	Random No.	l (cm)	$d > l$	$d \leq l$	Random No.	P.E.	Comp.
Exp. 1	50.0	0.234	29.0	*		0.208	*	
Exp. 2	50.0	0.906	1.97	*		0.716		*
	48.03	0.996	0.0802	*		0.600		*
	47.95	0.183	34.0	*		0.868		*
	13.95	0.351	20.9		*			

Fig. 1 Trajectories for a single layer

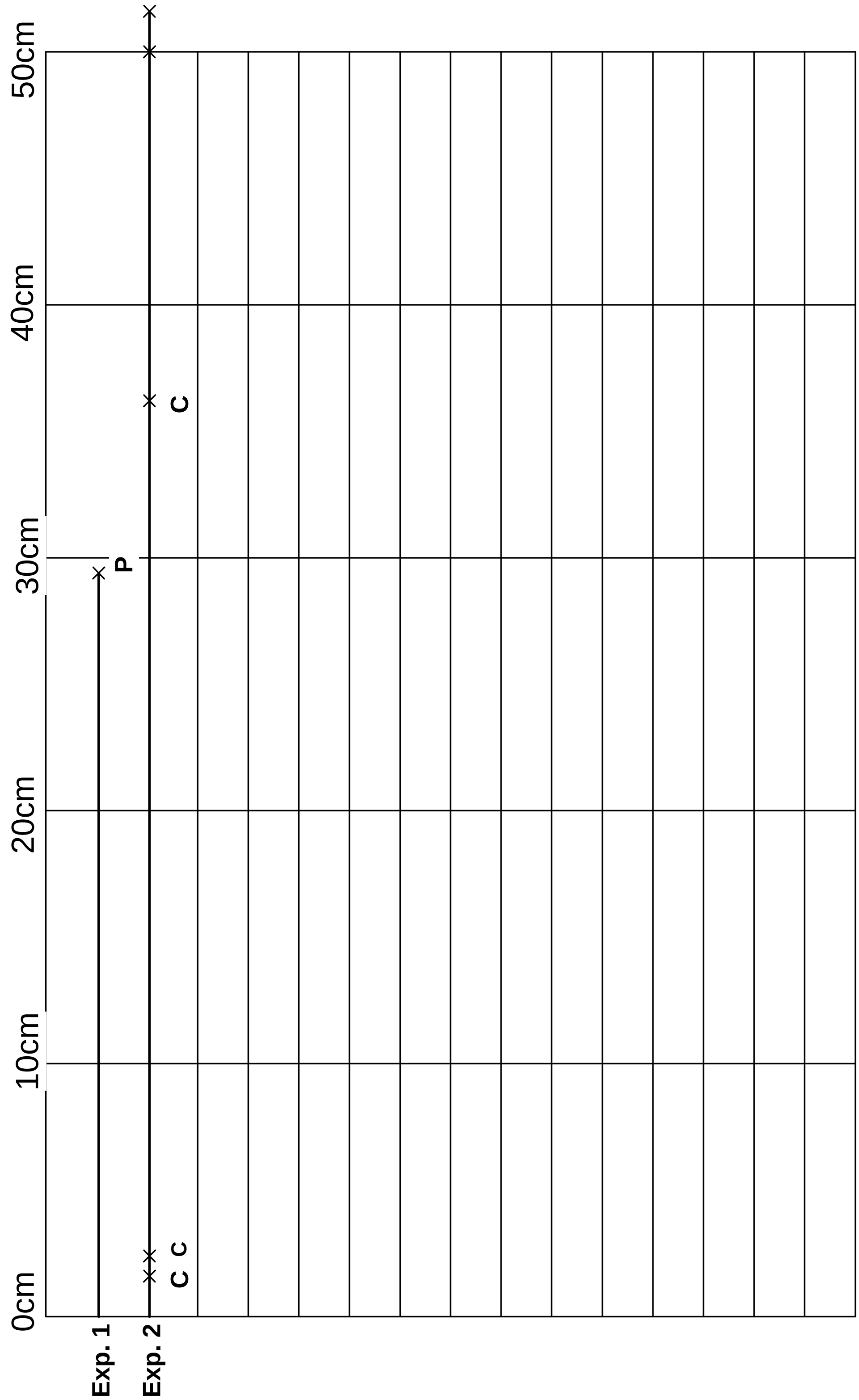
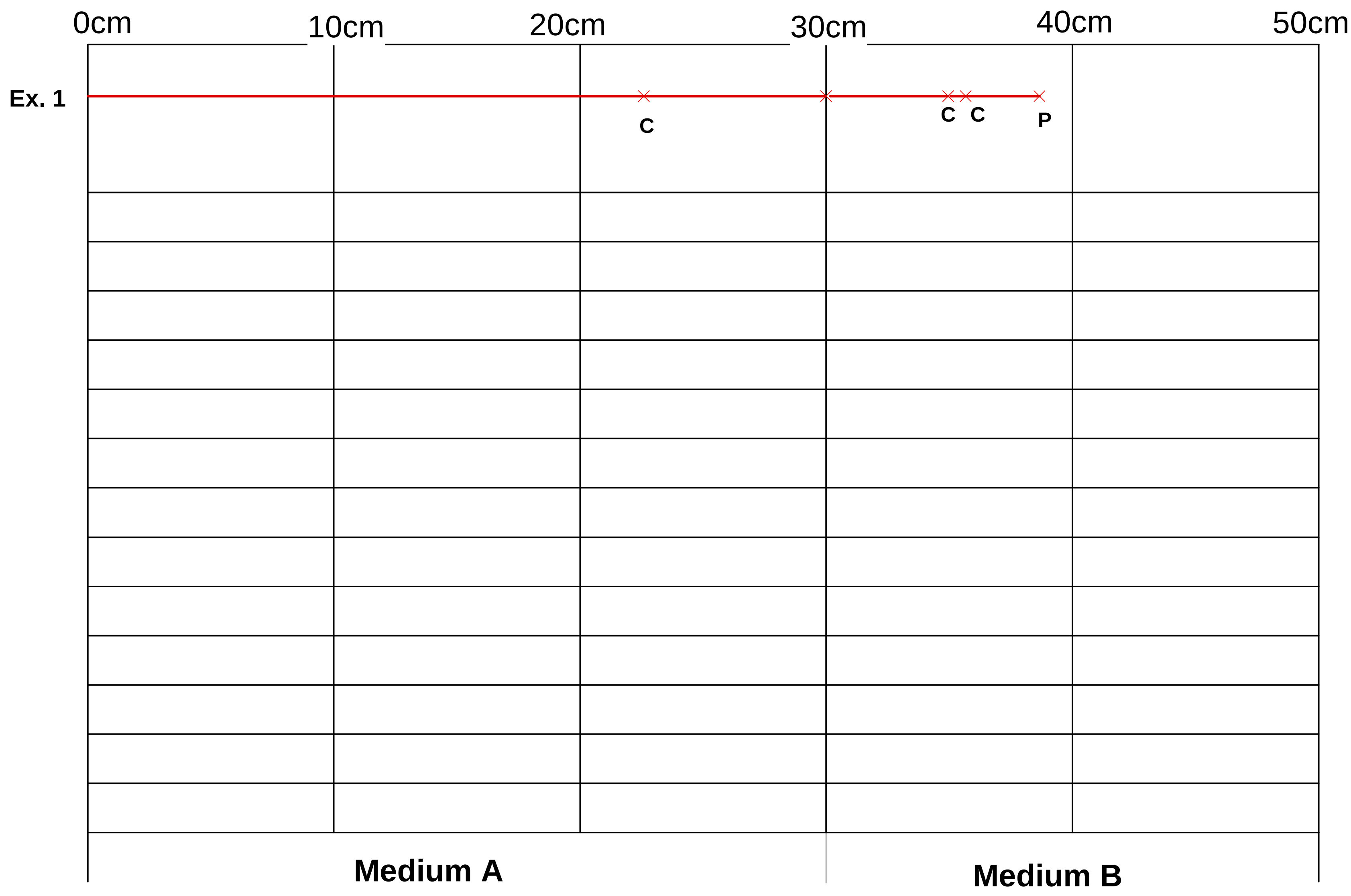


Table 4 Double Layer

	Medium A								Medium B							
No.	d(cm)	random number	l(cm)	$d > l$	$d \leq l$	random number	Photo	Compton	d(cm)	random number	l(cm)	$d > l$	$d \leq l$	random number	Photo	Compton
Ex. 1	30.0	0.329	22.2	○		0.612		○								
	7.8	0.234	29.0		○				20.0	0.281	3.81	○		0.906		○
									16.19	0.716	1.00	○		0.996		○
									15.19	0.600	1.53	○		0.183	○	

Fig. 2 Trajectories in double layers.



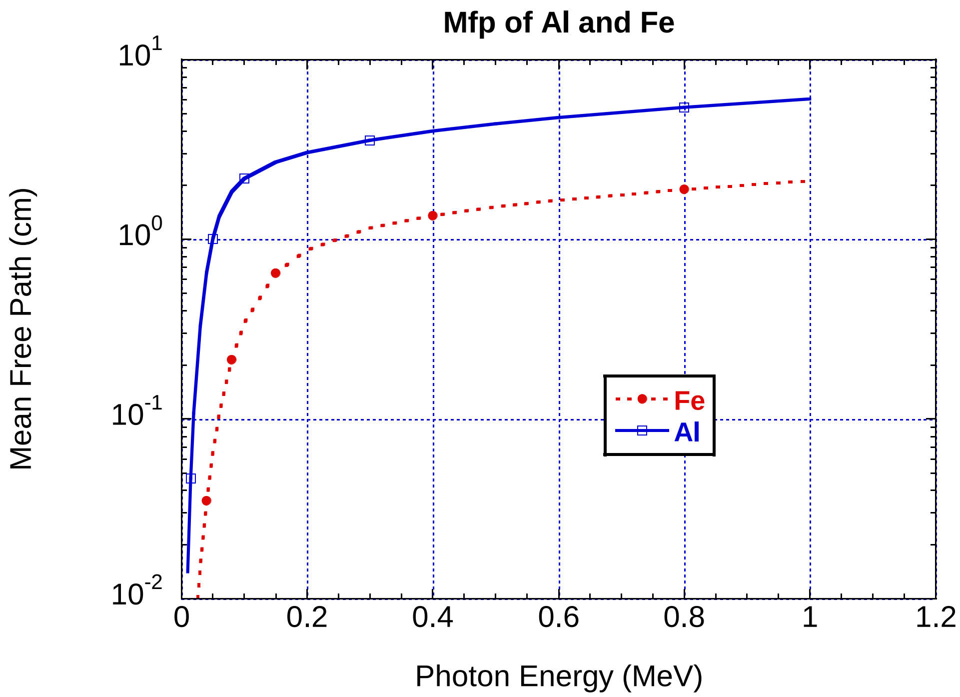


Figure 4: Mfp of Al and Fe as a function of the photon energy.

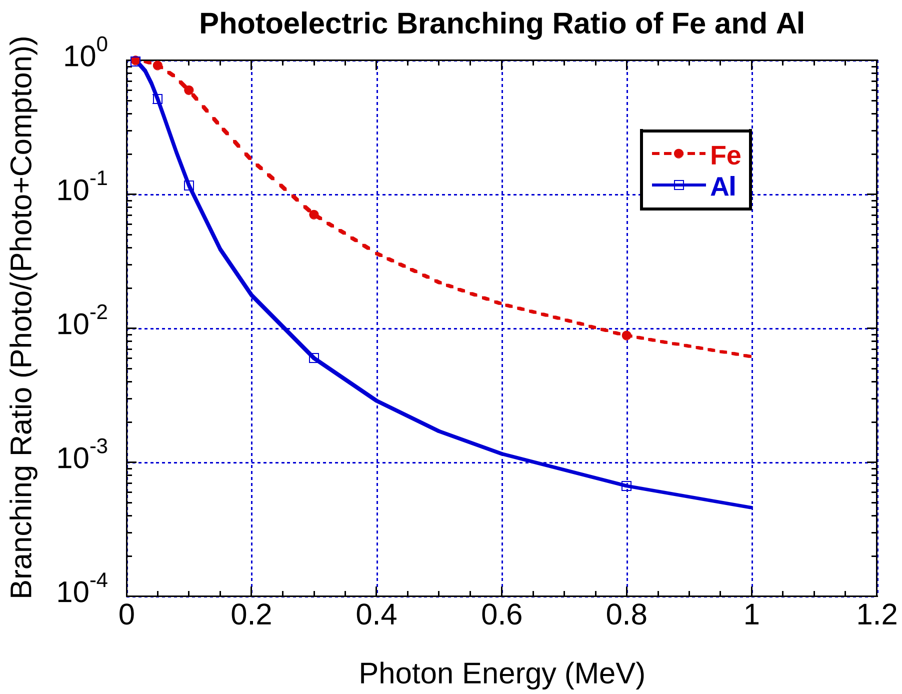


Figure 5: Photoelectric branching ratio of Al and Fe as a function of the photon energy.

Table 5

No.	E_0 (MeV)	Z (cm)	X (cm)	d (cm)	random number	l(cm)	$d > l$	$d \leq l$	random number	P.E.	Comp.	random number	Scatt.	E (MeV)	rabdom number	azim. angle
Source	0.5	0.0	0.0	10.0	0.351	4.33	○		0.259		○	0.572	90°	0.252	0.888	180°
Scatt. 1	0.252	4.33	0.0	∞	0.238	4.79	○		0.669		○	0.0478	0°	0.25		
Scatt.	0.25	4.33	-4.79	∞	0.949	0.175	○		0.324		○	0.579	90°	0.17	0.439	0°
Scatt. 3	0.17	4.33	-4.97	5.67												

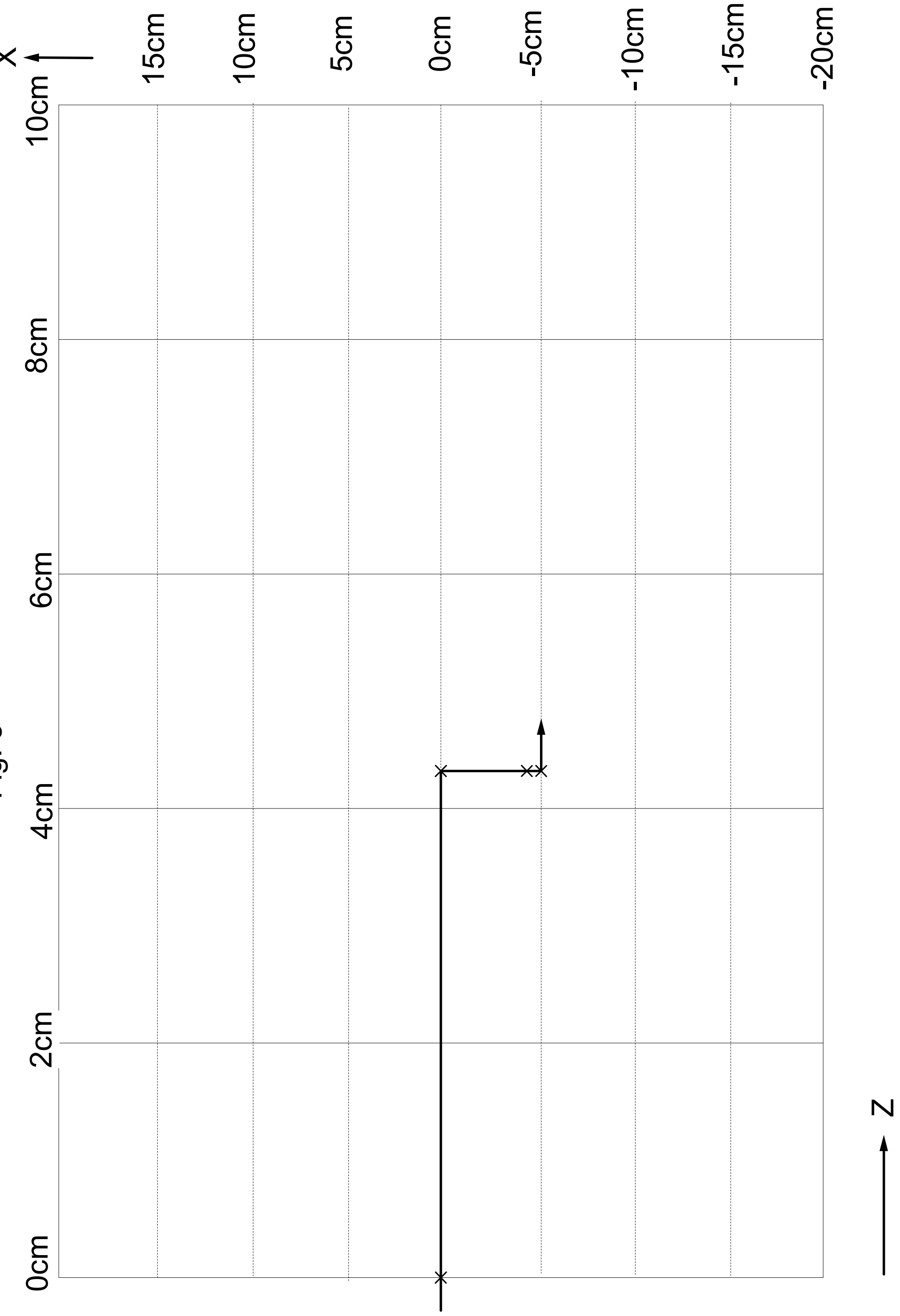

X
Z
15cm
10cm
5cm
0cm
-5cm
-10cm
-15cm
-20cm
10cm
8cm
6cm
4cm
2cm
0cm

Fig. 3

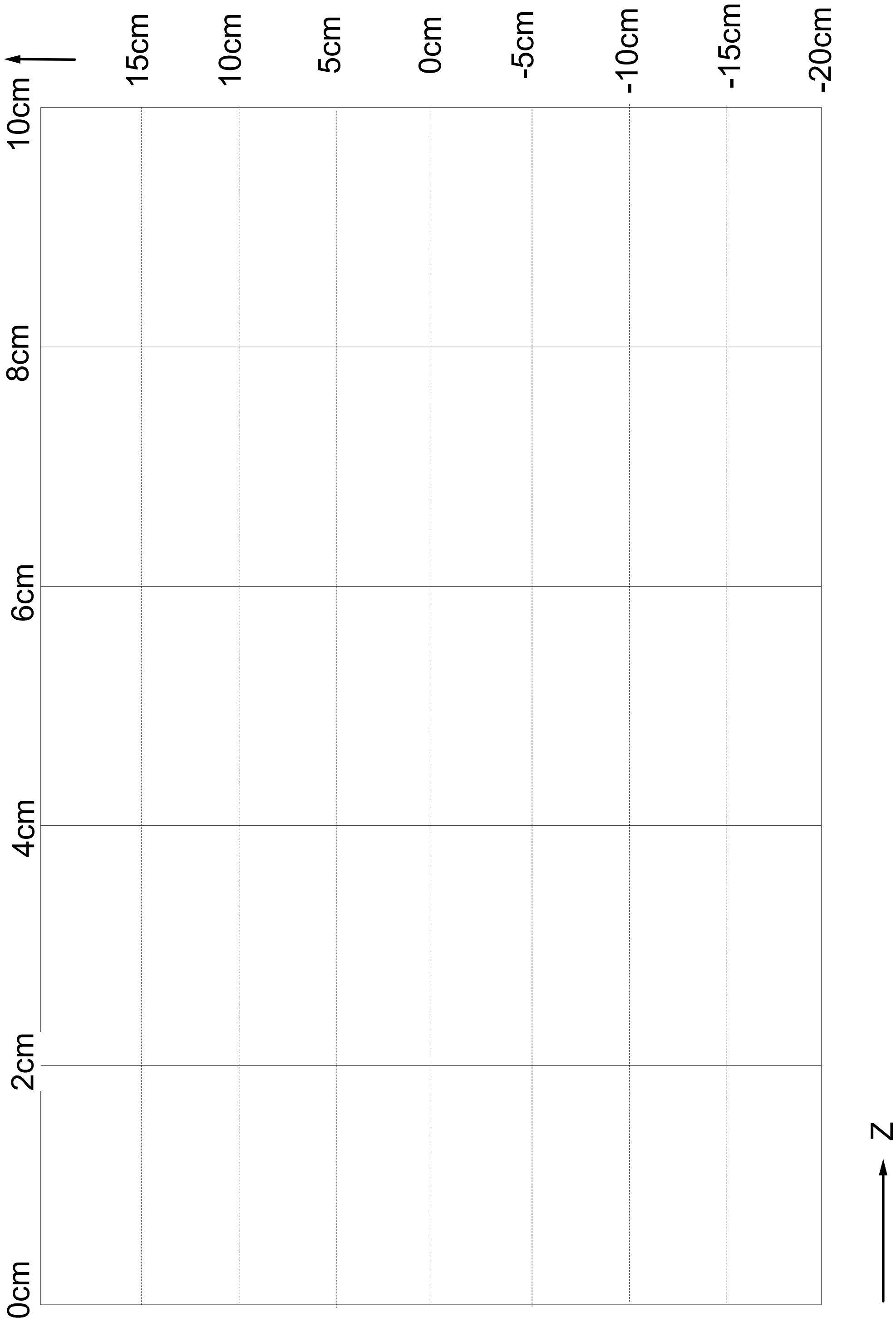

X
Z
10cm
8cm
6cm
4cm
2cm
0cm
15cm
10cm
5cm
0cm
-5cm
-10cm
-15cm
-20cm

Fig. 6

Table 6 Electron trajectories

Step n	Z_n	X_n	$0.0 < Z_n < 0.1$	E_n	$E_n > 0.01$	random number	θ	random number	ϕ
1	0.01	0.0	○	0.96	○	0.126	0°	0.983	180°
2	0.02	0.0	○	0.92	○	0.642	90°	0.081	0°
3	0.02	0.01	○	0.88	○	0.556	90°	0.817	180°
4	0.03	0.01	○	0.84	○	0.501	90°	0.920	180°
5	0.03	0.0	○	0.80	○	0.896	180°	0.618	180°
6	0.03	0.01	○	0.76	○	0.759	180°	0.690	180°
7	0.03	0.0	○	0.72	○	0.251	0°	0.094	0°
8	0.03	-0.01	○	0.68	○	0.371	90°	0.519	180°
9	0.02	-0.01	○	0.64	○	0.789	180°	0.567	180°
10	0.03	-0.01	○	0.60	○	0.397	90°	0.179	0°
11	0.03	0.0	○	0.56	○	0.576	90°	0.341	0°
12	0.02	0.0	○	0.52	○	0.517	90°	0.583	180°
13	0.02	0.01	○	0.48	○	0.909	180°	0.380	0°
14	0.02	0.0	○	0.44	○	0.326	0°	0.756	180°
15	0.02	-0.01	○	0.40	○	0.021	0°	0.132	0°
16	0.02	-0.02	○	0.36	○	0.132	0°	0.224	0°
17	0.02	-0.03	○	0.32	○	0.929	180°	0.646	180°
18	0.02	-0.02	○	0.28	○	0.019	0°	0.937	180°
19	0.02	-0.01	○	0.24	○	0.935	180°	0.530	180°
20	0.02	-0.02	○	0.20	○	0.117	0°	0.906	180°
21	0.02	-0.03	○	0.16	○	0.622	90°	0.074	0°
22	0.03	-0.03	○	0.12	○	0.886	180°	0.199	0°
23	0.02	-0.03	○	0.08	○	0.603	90°	0.164	0°
24	0.02	-0.04	○	0.04	○	0.763	180°	0.526	180°
25	0.02	-0.03	○	0.00	X				

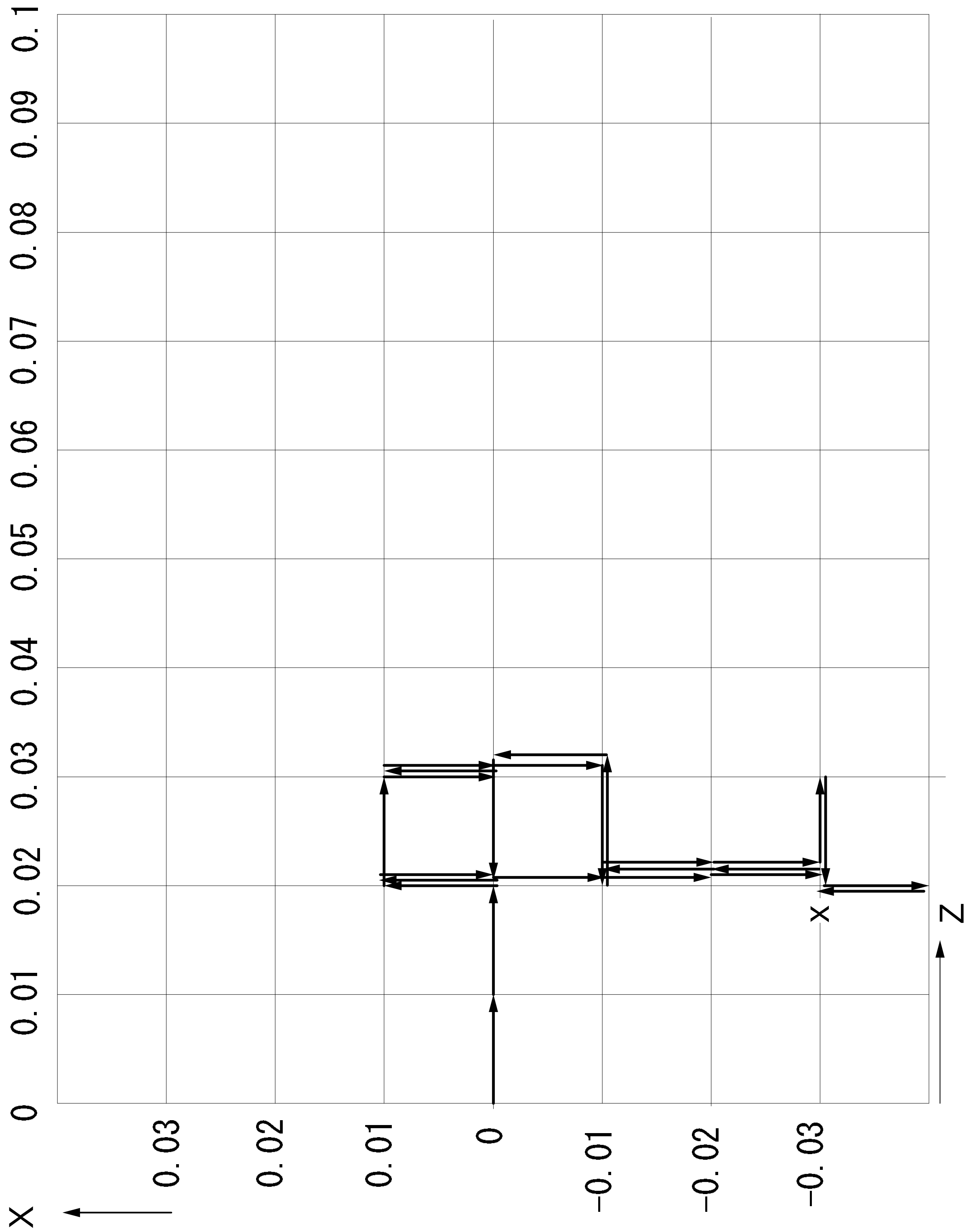
Table 7 Electron trajectories

Step n	Z_n	X_n	$0.0 < Z_n < 0.1$	E_n	$E_n > 0.01$	random number	θ	random number	ϕ
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									

Table 8 Electron trajectories

Step n	Z_n	X_n	$0.0 < Z_n < 0.1$	E_n	$E_n > 0.01$	random number	θ	random number	ϕ
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									

Fig. 7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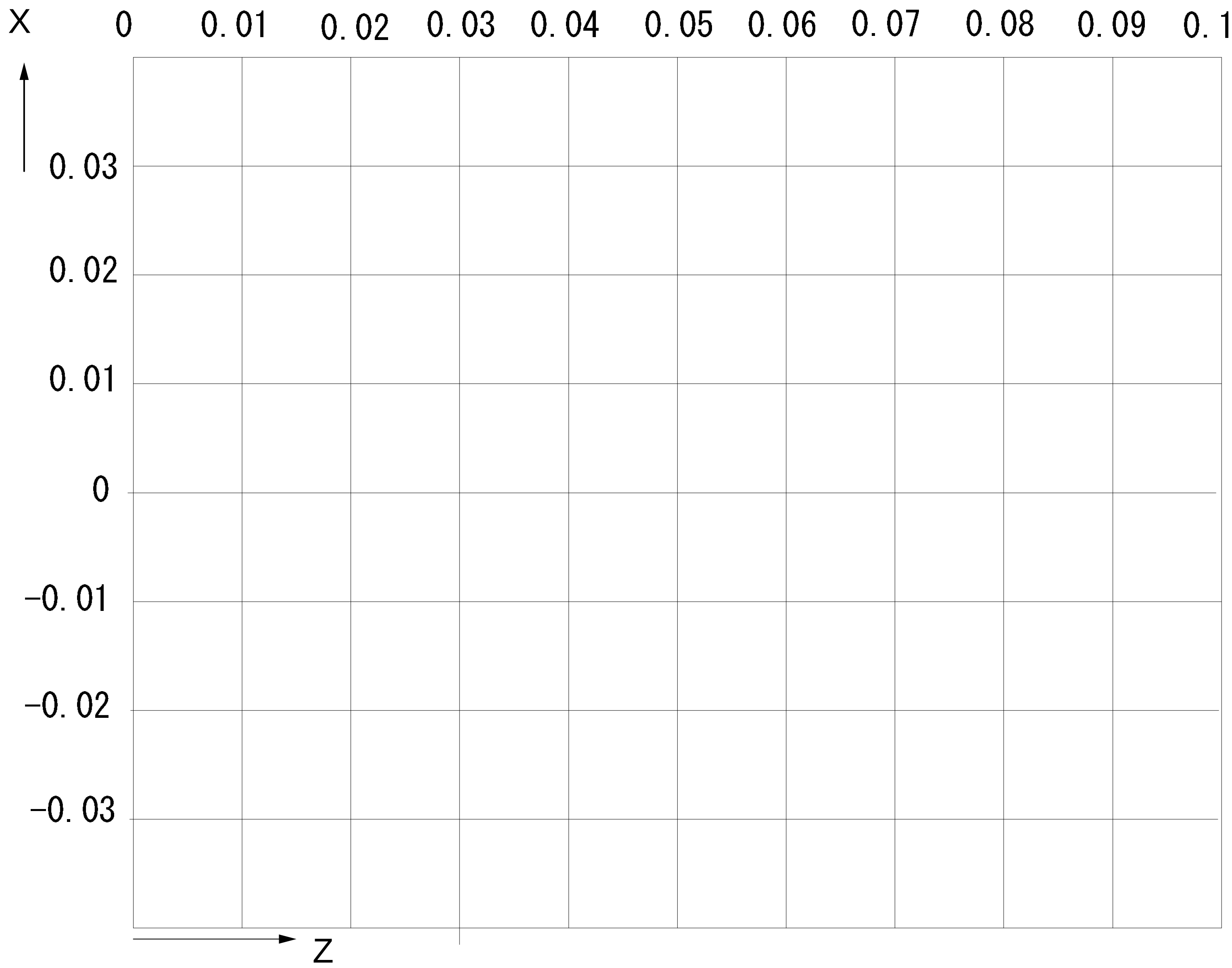
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
X
Z
x

Fig. 8


X
0
0. 01
0. 02
0. 03
0. 04
0. 05
0. 06
0. 07
0. 08
0. 09
0. 1
0. 03
0. 02
0. 01
0
−0. 01
−0. 02
−0. 03
Z

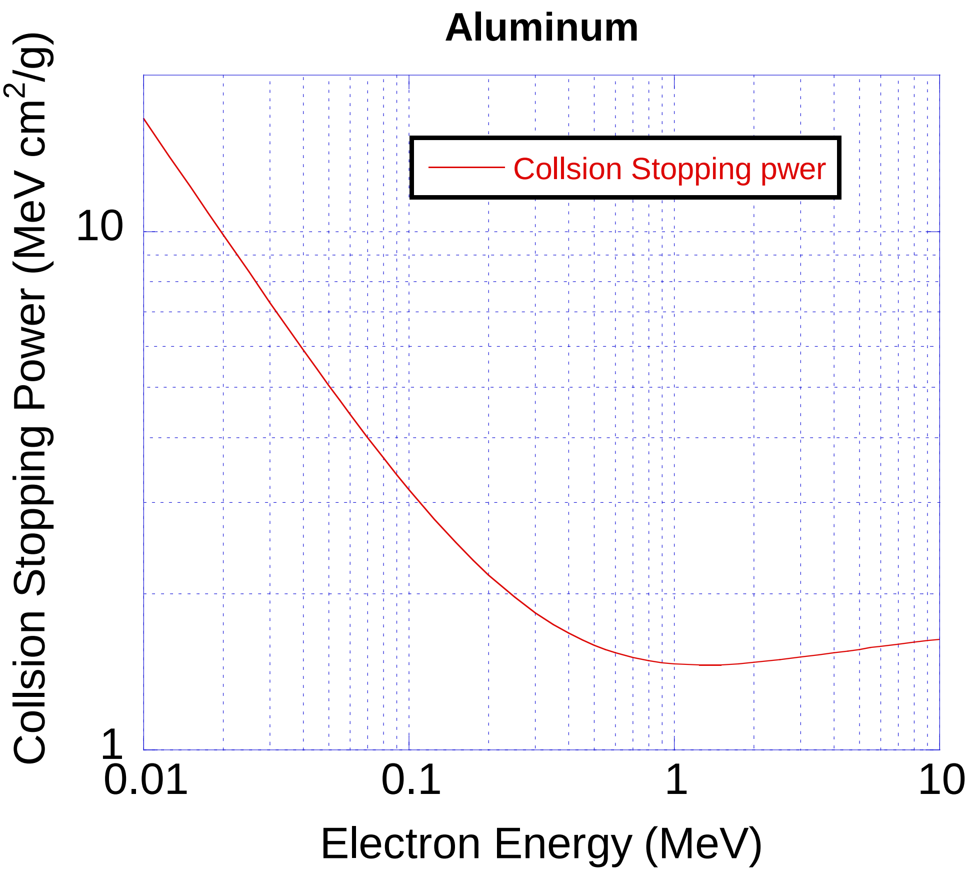
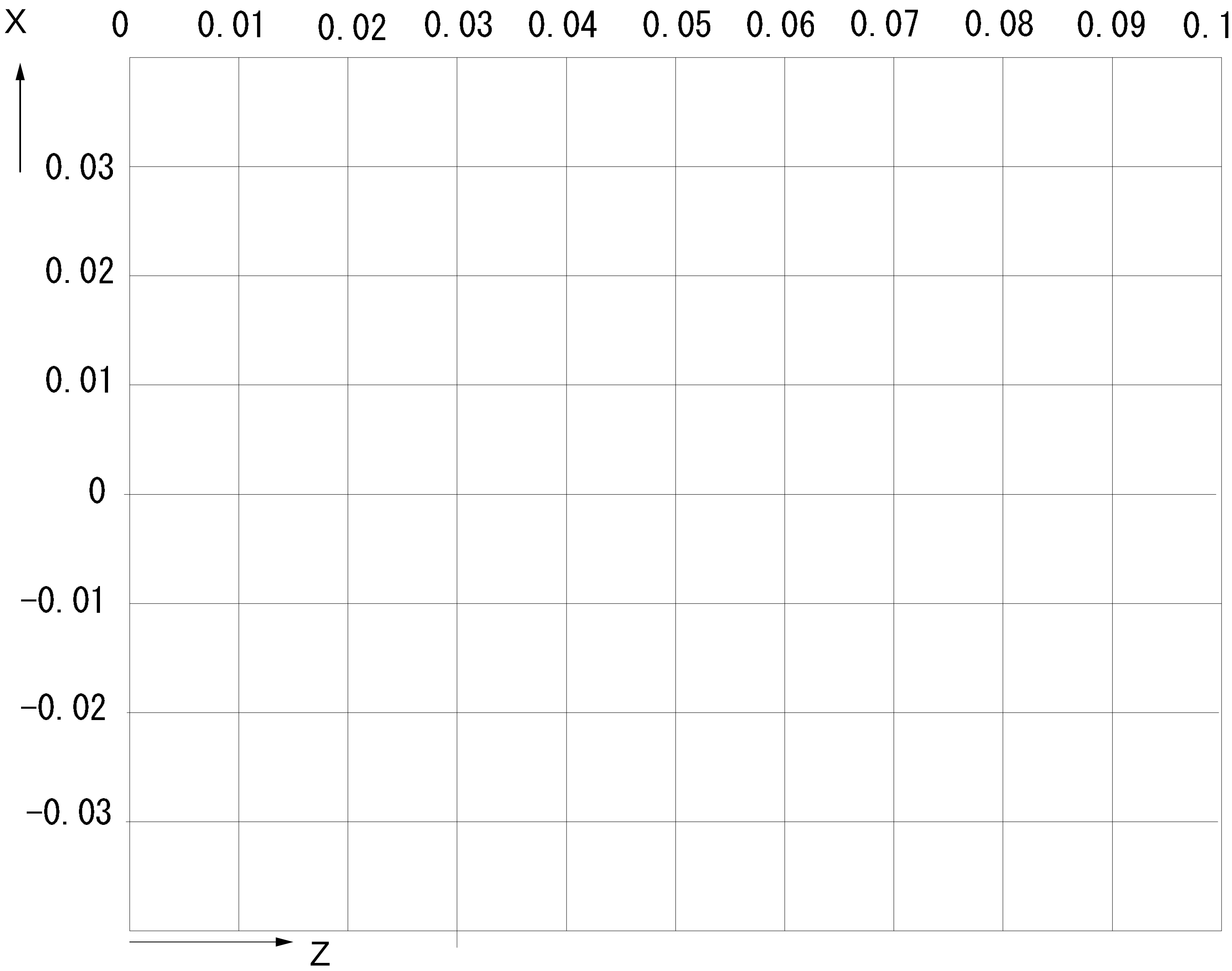


Figure 9: Collision stopping power of Al

Fig. 10


X
Z
0
0. 01
0. 02
0. 03
0. 04
0. 05
0. 06
0. 07
0. 08
0. 09
0. 1
0. 03
0. 02
0. 01
0
−0. 01
−0. 02
−0. 03