

Dynamic range of geometry size in CG

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We set a circle of radius R and center point $(0, y_0)$,

$$x^2 + (y - y_0)^2 = R^2, \quad (1)$$

as shown in fig. 1. Line A, $y = y_0 + R$, and the circle touch at $(0, y_0 + R)$. To estimate enhancement of numerical error near quadric surface, we evaluate position of crossing point when position of the line is slightly changed toward -y direction by Δy (Line B). What is a coordinate of cross point for the circle and Line B? (**Question 1**)

Answer 1 The equation of shifted line is,

$$y = y_0 + R - \Delta y. \quad (2)$$

By substituting eq. 2 into eq 1 and change x as Δx , we obtain,

$$\Delta x = \sqrt{2R\Delta y}. \quad (3)$$

Question 2 The value of $\Delta y/y$ is evaluated from presition of variable. In the case of double presition variable, the presition is 52 bits, or 2^{-52} . We can say that half of this value *i.e.* $2^{-53} = 1e-16$ is a typical numerical error for double presition variable. Derive Δx as a function of R and y_0 . Also, what is a ratio of $\Delta x/\Delta y$ if the center of the circle locates at origin point?

Answer 2

$$\Delta y = 10^{-16}(R + y_0 - \Delta y). \quad (4)$$

By subsituting eq. 4 into eq. 3, we obtain,

$$\Delta x = 1.4 \times 10^{-8} \sqrt{R(y_0 + R)}. \quad (5)$$

If the center of the circle locates at origin point, $y_0 = 0$. In this case, we obatin,

$$\Delta x = 1.4 \times 10^{-8} R. \quad (6)$$

By dividing eq. 6 by eq. 4, we obtain,

$$\Delta x/\Delta y = 1.4 \times 10^8. \quad (7)$$

This shows that a numerical error of line position is enhanced by a factor of 1.4×10^8 on circle or quadric surface if the position of the line is treated in double presition variable.

Question 3 We utilize small extra distance ϵ in CG to avoid numerical error. How can we determine an adequate value of ϵ ?

Answer 3 For a specified value of ϵ , we need to consider two conditions. First, ϵ should be larger than numerical error Δx (eq. 5 or eq 6). Second, to avoid numerical error, ϵ should be smaller than some fraction of geometry size. For example, we can tell this condition as " ϵ should be smaller than 0.01% of characteristic dimension (charD)". Then these two conditions are expressed as,

$$\Delta x < \epsilon < 0.01 \text{ charD}. \quad (8)$$

We can evaluate maximum of radium and minimum of chard for specified ϵ as is shown in Table 1. We may call them as dynamic range of geometry size in CG.

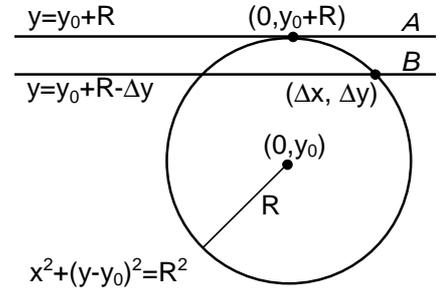


Figure 1: Cross point of a circle and a line.

ϵ	10^{-7}	10^{-6}	10^{-5}	10^{-4} *
Max of R (cm)	7	70	700	7×10^3
Min of chard (cm)	10^{-5}	10^{-4}	10^{-3}	10^{-2}

*:Default value