# A Monte-Carlo Method for Determining Absolute Scintillation-Photon Yields and Energy Resolution of Scintillators for Gamma Rays

H. Tawara, S. Sasaki, K. Saito<sup>1</sup>, and E. Shibamura<sup>2</sup>

High Energy Accelerator Research Organization
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

<sup>1</sup> The Graduate University for Advanced Studies
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

<sup>2</sup> College of Health Science, Saitama Prefectural University
820 Sannomiya, Koshiqaya, Saitama 343-8250, Japan

#### Abstract

The gamma-ray spectra from NaI(Tl) scintillation detectors have been studied quantitatively by a combination of experimental measurements and Monte-Carlo simulations. In this report, we describe the full-energy peak for an isotropic point source of 662-keV gamma rays measured with a NaI(Tl) crystal of 3-inch diameter by 3-inch long, as a typical case. We simulated three different kinds of spectra generated as results from gamma-ray interactions in NaI(Tl) and detection processes of scintillation photons with a photocathode: (1) an energy-deposition spectrum, (2) a scintillation-photon spectrum and (3) a photoelectron spectrum. The calculated photoelectron spectra were compared with those from experiments. Using the present method, the  $W_{\rm s}$  value, which is defined as the mean energy required to produce one scintillation photon corresponding to the full-energy deposition of the 662-keV gamma rays, was preliminary obtained to be 11.4 eV. Under the present experimental condition, the total energy resolution of the full-energy peak can be explained in terms of an intrinsic energy resolution caused by the non-linear energy response of NaI(Tl) for electrons, the transfer variance of the scintillation-photon number in the detector system, the photoelectron statistics and electric noise. These resolution losses were also estimated quantitatively.

#### 1 Introduction

Scintillation detectors are widely used for gamma-ray spectroscopy. However, the precise measurements of absolute scintillation efficiency of various scintillators for gamma rays are difficult, because the energy deposition due to gamma rays in the scintillators, and the transport and detection of scintillation photons in a scintillation detector system are quite complicated processes to analyze quantitatively.

Our study has two objectives: (1) to determine the mean energy required to produce one scintillation photon ( $W_s$  value) of various scintillators for radiation, and (2) a quantitative understanding of the energy resolution of a scintillation detector system. In the present study, the  $W_s$  value is defined for gamma-rays having monochromatic energy:

$$W_{\rm s} = \frac{T}{N_{\rm s}}$$
 (eV), (1)

where T is the energy of incident gamma rays in eV and  $N_s$  is the mean number of scintillation photons produced in the scintillator by the full-energy deposition of the gamma rays. A detailed description of the  $W_s$  value for gamma rays is given elsewhere[1].

The method used in our study is a combination of experimental measurements and Monte-Carlo simulations. Monte-Carlo simulations have been conducted with two computer codes, EGS4[2] and SPC[1]. These codes have been improved continuously and now incorporate experimental and theoretical data indispensable for analyzing of the gamma-ray spectrum measured by NaI(Tl) scintillation detectors: generation of photoelectrons, characteristic X-rays and Auger electrons in K- and L-shell photoelectric effects of Na, I and Tl; energy dependence of the scintillation response for electrons in an energy region from 1 keV to 1 MeV; the quantum efficiency of a photocathode, which is coupled with the NaI(Tl) crystal, and its non-uniform distribution. These Monte-Carlo codes allow us to determine the gross conversion efficiency of the NaI(Tl) scintillation detector system from scintillation photons to photoelectrons. In addition, they can be used to estimate the individual sources of the energy resolution losses of the detector system.

In the following sections, we treat the gamma-ray spectrum, which is obtained by a NaI(Tl) crystal (3 inch diameter  $\times$  3 inch long) irradiated with an isotropic point gamma-ray source of <sup>137</sup>Cs. As a preliminary result, we show the  $W_s$  value corresponding to the 662-keV full-energy peak. In addition, it is clarified that the main sources of the energy resolution losses of the full-energy peak are the intrinsic energy resolution, transfer variance, photoelectron statistics and electric noise, under the present experimental conditions.

## 2 Method for analyzing the gamma-ray spectra

In the case of a scintillator exposed to gamma rays, the scintillation photons are generated as a result from energy deposition due to secondary electrons, such as photoelectrons, Compton-recoil electrons and electron-positron pairs. When the scintillation response for electrons (electron response) depends on electron energy, the gamma-ray spectrum expressed in scintillation-photon number (scintillation-photon spectrum) becomes deformed from the original one, expressed in energy deposition (energy-deposition spectrum). In general, this scintillation-photon spectrum cannot be observed directly. When the scintillator is coupled with a photomultiplier (PMT), the scintillation photons are converted into photoelectrons by the photocathode of the PMT. Therefore, the gamma-ray spectrum can be observed as counts per unit photoelectron number (photoelectron spectrum).

The method for analyzing these three kinds of gamma-ray spectra consists of three parts: (1) Monte-Carlo simulations of gamma-ray interactions with scintillation materials and the generation of scintillation photons, by the EGS4 code incorporating the electron response data of the scintillator; (2) Monte-Carlo simulations of Scintillation-Photon Transport (SPT) inside the scintillation detector system and the generation of photoelectrons at the photocathode by the SPC code incorporating the quantum-efficiency data of the photocathode; and (3) experimental measurements of the absolute number of photoelectrons emitted from the photocathode.

In the case of a scintillator coupled with a photomultiplier, the number of photoelectrons,  $N_{pe}$ , is described in terms of  $N_s$  and four parameters,

$$N_{\rm pe} = N_{\rm s} F_{\rm c} Q_{\rm e} F_{\rm s} F_{\rm g},\tag{2}$$

where  $F_c$  is the collection efficiency of the detector system for scintillation photons to the photocathode,  $Q_e$  is the quantum efficiency of the photocathode,  $F_s$  is the collection efficiency of a collection electrode for the photoelectrons and  $F_g$  is a total gain of a PMT. Thus,  $N_s$  in eq.(1) can be estimated by

$$N_{\rm s} = \frac{N_{\rm pe}}{F_{\rm c}Q_{\rm e}F_{\rm s}F_{\rm g}}.$$
 (3)

# 3 Experimental measurement of $N_{\text{pe}}$

Fig. 1 shows a schematic diagram of an experimental apparatus. The NaI(Tl) scintillator is irradiated with an isotropic gamma-ray sources of <sup>137</sup>Cs. The scintillation photons are converted into

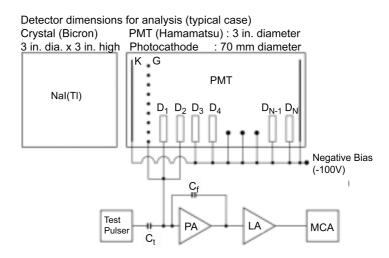


Figure 1: Apparatus for photoelectron measurements.

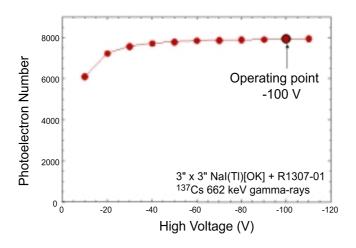


Figure 2: Saturation curve of the full-energy peak measured by the PMT in the PD-mode.

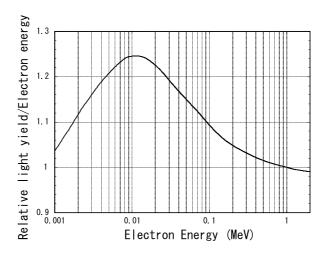


Figure 3: Electron-response data[12] of NaI(Tl) used for the present simulations.

photoelectrons with the photocathode (K). In order to measure the precise number of photoelectrons, the PMT is operated in the Photo-Diode mode (PD mode). A grid (G), the first dynode (D1) and the second dynode (D2) are connected together and act as a collector electrode for those photoelectrons emitted from the photocathode. The photocathode, the other dynodes and an anode (A) are connected together and act as a negative-bias electrode. The charge signals from the collector electrode are fed into a charge-sensitive preamplifier. The electric circuit including the charge-sensitive preamplifier is calibrated with a high-precision pulser in order to obtain the absolute number of photoelectrons collected by the collector electrode. The details of experiments have been described in the previous publications [3, 4, 5, 6, 7].

Figure 2 shows a saturation curve of  $N_{\rm pe}$  corresponding to the full-energy peak in the PD-mode. This saturation curve indicates that the photoelectrons are fully collected with the collector electrode around -100 V and multiplication of photoelectrons does not take place. Therefore, both  $F_{\rm g}$  and  $F_{\rm s}$  are unity under the present experimental conditions.

#### 4 Monte-Carlo simulations

The EGS4 code was used to score the secondary electrons generated from gamma-ray interactions in NaI(Tl), which are photoelectrons, Compton recoil electrons, electron-positron pairs and Auger electrons. In order to treat low-energy photon transport precisely, the following items were incorporated into the default EGS4 code[8, 9, 10, 11]: photoelectrons from the K-, L1-, L2- and L3-shell photoelectric effects, K- and L-X rays, energy deposition due to M- and higher-shell photoelectric effects, K- and L-Auger electrons, and Doppler broadening of Compton-scattered photons. The secondary-electron transport was not simulated for reducing computation time. The energy of each secondary electron is converted into scintillation photons using the relative electron-response in Fig. 3, which was measured by Rooney et al.[12]. The electron response of NaI(Tl) is non-linear in an energy region less than 1 MeV, as shown in Figure 3.

Seven different processes relating to SPT are incorporated into the present SPC code, as shown in Fig. 4. The SPC code was originally developed by our group and recently linked to an EGS4 usercode. The details of the Monte-Carlo simulations are described in a separate paper[1]. In the present SPT simulations for NaI(Tl) detectors, we assumed following: the reflectivity of a reflector for scintillation photons is 0.975; the reflection of scintillation photons on the reflector is diffusive (directions of scintillation photons after reflection is sampled randomly in the simulation); Snell's law is valid at the interface between NaI(Tl) and a crystal window; the refractive indices of NaI(Tl) and the crystal windows of the scintillator and the PMT are 1.47 and 1.85, respectively; and scintillation-light

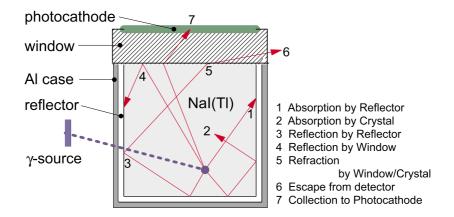


Figure 4: Processes treated in a scintillation photon transport (SPT) simulation.

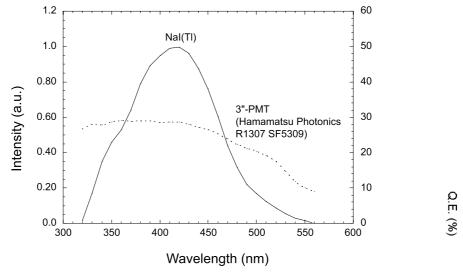


Figure 5: Emission spectrum of NaI(Tl) and quantum efficiency (Q.E) of photocathode.

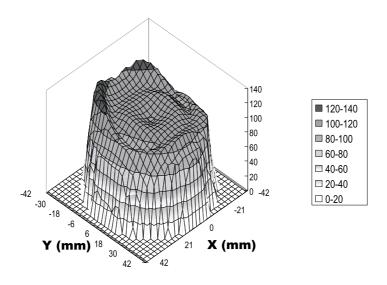


Figure 6: X, Y dependence of Q.E.

attenuation inside NaI(Tl) is negligible.

The photocathode conversion efficiency from photons to photoelectrons is generally called the Quantum Efficiency  $(Q_e)$ . The absolute value of  $Q_e$  in a central area of 4 x 4 cm<sup>2</sup> of the photocathode used was measured as a function of wavelength by the manufacturer of the PMT (Hamamatsu Photonics), which is shown in Fig. 5, along with the emission spectrum of NaI(Tl). Figure 6 shows the relative X- and Y-distribution of  $Q_e$ , which was measured with a collimated light source. From the data shown in Figs. 5 and 6, the averaged  $Q_e$  of the photocathode was estimated to be 0.272. The non-uniformity data of the photocathode was also incorporated into the SPT simulation.

#### 5 Results and discussions

Figure 7 shows the results from Monte-Carlo simulations. The energy-deposition spectrum obtained from the EGS4 simulation indicates a monochromatic full-energy peak at the right endpoint of the spectrum, as shown in Fig. 7 (a). The non-linearity of the electron response broadens the full-energy peak in the scintillation-photon spectrum, and causes fine structures on this peak[13], as shown in Fig. 7 (b). The photoelectron spectrum obtained from an SPT simulation following the EGS4 simulation is also shown in Fig. 7 (b). In measuring the photoelectron spectrum, the resolution loss due to electric-noise ( $\sigma_{\text{noise}}$ ) was 470 electrons (4.4%) in FWHM. In order to compare the simulated spectrum with the measured one, the photoelectron spectrum in Fig. 7 (b) was obtained as a convolution with Gaussian functions having a standard deviation of (470/2.35=) 200 electrons.

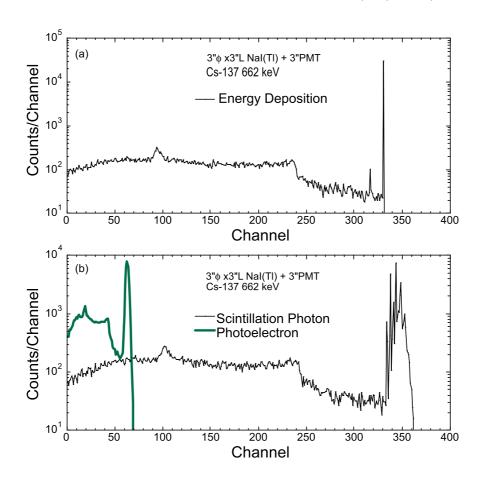


Figure 7: Monte-Carlo simulations. (a) The energy-deposition spectrum. (b) The scintillation-photon spectrum (thin line) and the photoelectron spectrum (thick line).

Figure 8 is a comparison of the photoelectron spectra. This figure shows that the full-energy

peak from the simulation agrees very well with that from the measurement. Although these spectra are different around their back-scattered peaks, it can be explained from a lack of massive materials surrounding NaI(Tl), such as the PMT etc., in the present geometry for the EGS4 simulation. The total energy resolution in FWMM of the full-energy peak ( $\sigma_{\text{total}}$ ) was 7.7% in the measurement and 7.6% in the simulation.

The gross conversion efficiency from scintillation photons to photoelectrons, which corresponds to  $F_c \times Q_e$ , was estimated to be 0.182 from a comparison of the full-energy peaks of the scintillation-photon spectrum and of the photoelectron spectrum, in Fig. 7 (b). The measured full-energy peak in Fig. (8) was estimated to be about 10600 electrons from fitting with Gaussian functions. Therefore, the mean number of scintillation photons produced by the full-energy deposition of 662-keV gamma rays was calculated to be 58200 from eq.(3). We can thus obtain 11.4 eV as the preliminary value of the  $W_s$  from eq.(1).

The energy resolution expected from photoelectron statistics ( $\sigma_{pe}$ ) was calculated to be 2.3% from 10600 electrons. However, this value is too small to explain the measured total resolution of 7.7%, even if the resolution loss due to electric noise of 4.4% is taken into account. The other main sources of the resolution loss are an intrinsic resolution ( $\sigma_{\rm int}$ ) and a transfer variance ( $\sigma_{\rm trans}$ ). Figure 9 shows a broadening of the full-energy peak (the intrinsic resolution) in the scintillation-photon spectrum due to a difference in the gamma-ray interaction and the non-linear electron response. From the standard deviation of this distribution, the intrinsic resolution was calculated to be 3.5% in FWHM. If the NaI(Tl) crystal has some defects, such as an inhomogeneity of Tl activators, they would cause additional resolution losses. However, such effects on the energy resolution were not treated in the present study. The transfer variance consists of the following sources: (1)scintillation-photon collection loss of a scintillation detector system, (2) photoelectron conversion loss of the photocathode, (3) nonuniformity of the quantum efficiency and (4) photoelectron collection loss. In the case of 662-keV gamma-ray irradiation, the scintillation photons are generated almost uniformly over the entire volume of the NaI(Tl) scintillator. In order to estimate the transfer variance, we simulated the distribution of a collected number of scintillation photons under a condition similar to the present case. Namely, the location of isotropic scintillation sources was randomly sampled inside a 3-inch-diameter, 3-inch-long scintillation crystal; each source generated 60000 scintillation photons. As a result of the simulation,  $\sigma_{\rm trans}$  was estimated to be 4.6% in FWHM.

If these sources of resolution loss are independent each other, we can calculate the total energy resolution from the following relation:

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{int}}^2 + \sigma_{\text{trans}}^2 + \sigma_{\text{pe}}^2 + \sigma_{\text{noise}}^2}.$$
 (4)

Using eq.(4) and values described above, the total resolution was calculated to be 7.6%, which agrees well with the results of the measurement and the simulation.

### 6 Conclusions

The 662-keV gamma-ray spectra from NaI(Tl) scintillation detectors (crystal size,  $3"\phi \times 3"$ ) were analyzed quantitatively by a combination of experimental measurements and Monte-Carlo simulations. The  $W_s$  value corresponding to the full-energy deposition of the 662-keV gamma rays was preliminary determined to be 11.4 eV. The total energy resolution of the full energy peak in FWHM was obtained to be 7.7% in the experiment and 7.6% in the simulations. This total resolution consists of the intrinsic energy resolution (3.5%) caused by the non-linear energy response of NaI(Tl) for electrons, the transfer variance (4.6%) of the scintillation-photon number in the detector system, the photoelectron statistics (2.3%) and the electric noise (4.4%). It is considered that the present method will help to analyze different types of scintillation detectors in the future.

# Comparison of Phtoelectron Spectra

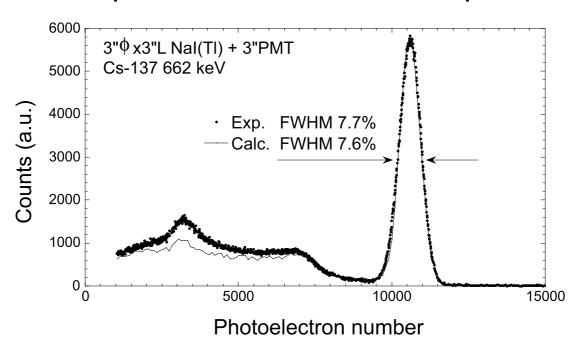


Figure 8: Comparison of the photoelectron spectra.

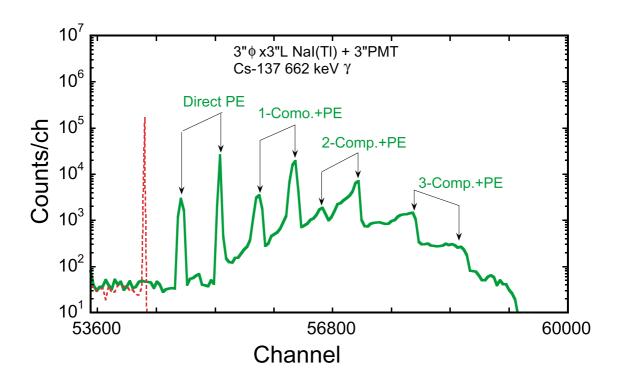


Figure 9: Intrinsic crystal resolution.

## References

- [1] H. Tawara, S. Sasaki, K. Saito, E. Shibamura and M. Miyajima, KEK Proceedings **99-15**, 44 (1999).
- [2] W. R. Nelson, H. Hirayama and D. W. O. Rogers, SLAC-265 (Stanford University, Stanford 1985).
- [3] M. Miyajima, S. Sasaki and E. Shibamura, Nucl. Instrum. Meth. 224(1984)331.
- [4] M. Miyajima, S. Sasaki, H. Tawara, and E. Shibamura, IEEE Trans. Nucl. Sci. 39, 536 (1992).
- [5] M. Miyajima, S. Sasaki and H. Tawara, IEEE Trans. Nucl. Sci. 40(1993)417.
- [6] S. Sasaki, H. Tawara, and M. Miyajima, KEK Proceedings 93-8, 20 (1993).
- [7] E. Shibamura, S. Sasaki, H. Tawara and M. Miyajima, KEK Proceedings 99-8, 175 (1999).
- [8] Y. Namito, S. Ban and H. Hirayama, Radiat. Phys. and Chem. 53(1998)283.
- [9] H. Hirayama, Y. Namito and S. Ban, KEK Internal **96-10** (1996).
- [10] Y. Namito, S. Ban and H. Hirayama, Nucl. Instrum. Meth. A349(1994)489.
- [11] Y. Namito, S. Ban and H. Hirayama, Nucl. Instrum. Meth. A332(1993)277.
- [12] B. D. Rooney and J. D. Valentine, IEEE Trans. Nucl. Sci. 44(1997)509.
- [13] J. D. Valentine, B. D. Rooney and J. Li, *IEEE Trans. Nucl. Sci.* 45(1998)512.