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A Convolution Method for Determining Temperature Rise in Targets Struck by Beams of Various Size [1]

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Abstract

The temperature rise in targets struck by high-energy electrons can be calculated using the EGS4 code[1] simply by scoring the energy-deposition density in small cylindrical volumes centered upon, and divided along, the direction of the beam. The temperature rise per pulse, ΔT_p (°C/pulse), is then obtained for each volume using the specific heat of the material, and the time dependence of the heat flow can be calculated using conventional heat-transfer principles. Most typically the beam size is accounted for in a straight-forward way by sampling the incident coordinates, but this involves yet another statistical process that can result in a significant increase in computation time in order to reduce the variance, particularly for thick targets at very high energies. In this paper an off-line convolution method is presented in which the symmetry of the geometry and the Gaussian shape of the beam is used, along with a set of EGS4 runs made with a δ-function (*i.e.,* pencil) beam, to quickly obtain the temperature rise on the pulse for beams of any size. Examples are given for studies that have recently been performed at SLAC in the design of the Next Linear Collider.

1 Introduction

There are three important quantities which must be determined when designing targets capable of handling large temperature-rise excursions. Namely, the temperature rise on the pulse, the maximum stress at the central core and the steady-state temperature. The latter two, however, are derivable from the temperature rise per pulse, ΔT_p (°C/pulse), and this, in turn, is obtained from the energy-deposition density (*i.e.,* fractional energy-loss per unit volume), $dE/E_0 dV$, as follows[2] [3] [4] :

$$\Delta T_p = C \frac{N E_0}{\rho C_p} \frac{1}{E_0} \frac{dE}{dV} \ , \tag{1}$$

where

$$\begin{aligned} \rho &= \text{material density (g/cm}^3\text{)}, \\ C_p &= \text{heat capacity} \approx \frac{6.0}{A} \text{ (cal/g}^\circ\text{C)}, \\ N &= \text{electrons/pulse}, \\ E_0 &= \text{incident beam energy (MeV)}, \\ A &= \text{atomic weight (g/mole)}, \\ C &= 1.6 \times 10^{-13}/4.184 \text{ (cal/MeV)}. \ . \end{aligned}$$

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The maximum radial stress (r=0) in a pulse, σ_r (psi), is then obtained with

$$\sigma_r = \frac{\alpha E_y \Delta T_p}{2(1-\nu_p)}, \qquad (2)$$

where

$$\begin{aligned} \nu_p &= \text{Poisson ratio (0.25 to 0.30)}, \\ \alpha &= \text{coefficient of thermal expansion } (^\circ\text{C}^{-1}), \\ E_y &= \text{Young's modulus (psi)} . \end{aligned}$$

The steady state temperature profile, $T(r)$, is given by equating the heat input, $\dot{Q}$, inside a cylinder of radius r to the heat conduction through the surface of the cylinder.

$$\dot{Q} = \nu N E_0 C \int_0^r \frac{1}{E_0}\frac{dE}{dV} 2\pi r\, dr = k_T 2\pi r \frac{dT}{dr}, \qquad (3)$$

where

$$\begin{aligned} \nu &= \text{pulse repetition rate (sec}^{-1}), \\ k_T &= \text{thermal conductivity coefficient (cal sec}^{-1}\text{ cm}^{-1}\ {}^\circ\text{C}^{-1}) . \end{aligned}$$

Therefore, one only needs to determine dE/E_0dV using the EGS4 code.

2 Accounting for the Radial Extent of the Beam

The straight-forward method of accounting for the beam size with EGS4 is to sample the incident coordinates (X_I, Y_I) over an appropriate distribution, such as a Gaussian, just prior to each call to `SUBROUTINE SHOWER`. However, several problems arise from this direct method. First of all, at high energies (multi-GeV) and for thick targets (many radiation lengths), a large amount of computer time is spent just tracking the particles in the shower to the very low energies required in determining the energy-deposition density itself. A limit must then be imposed on the number of incident particles that can be sampled, for any given beam size, and the end result is that the radial distribution for the temperature rise contains large fluctuations.

Secondly, the calculation has to be run over and over again for each of the beam-spot sizes of interest. Clearly, the time is better spent performing EGS4 calculations with good statistics, using a $\delta(X_I)\delta(Y_I)$ (*i.e.*, pencil beam) input, and subsequently performing convolution-type integrations for each of the beam sizes of interest.

The convolution method that we have developed applies to round Gaussian beams—*i.e.*, $\sigma_x = \sigma_y = \sigma$. It was developed by Ecklund and Nelson in 1981 [5] and subsequently used in the thesis by Donahue [6] .

3 Gaussian Convolution of Pencil Beams

The general form of a one-dimensional Gaussian distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-\frac{(x-\overline{x})}{2\sigma^2}}\, d\overline{x} \ . \qquad (4)$$

If we assume that the energy-deposition density of the pencil beam is given by

$$W_0(r) \equiv \frac{1}{E_0}\frac{dE}{dV} \ , \qquad (5)$$

then the convoluted energy-deposition density is

$$W_\sigma(r) = f * W_0 = \int f(x) W_0(x)\, dx \ . \tag{6}$$

For a two-dimensional Gaussian distribution in radial coordinates ($\overline{x} = \overline{r}\cos\overline{\theta}$, $\overline{y} = \overline{r}\sin\overline{\theta}$),

$$\begin{aligned} W_\sigma(r) &= \frac{1}{2\pi\sigma^2}\int_0^\infty d\overline{r}\int_0^{2\pi} d\overline{\theta}\,\overline{r}e^{-(\frac{r^2+\overline{r}^2-2r\overline{r}\cos(\theta-\overline{\theta}))}{2\sigma^2}} W_0(\overline{r}) \\ &= \frac{1}{2\pi\sigma^2}\int_0^\infty \overline{r}d\overline{r}\, e^{\frac{-(r-\overline{r})^2}{2\sigma^2}} W_0(\overline{r})\int_0^{2\pi} d\overline{\theta}\, e^{-\frac{r\overline{r}}{\sigma^2}(1-\cos\overline{\theta})} \ , \end{aligned} \tag{7}$$

where we have taken $\theta = 0$ without loss of generality.

Since the EGS4 output is in the form of a histogram averaged over radial bins, we have

$$W_\sigma(i) \equiv \int_{r_i}^{r_{i+1}} W_\sigma(r)\, r\, dr \ . \tag{8}$$

The convoluted distribution with the same binning is then

$$W_\sigma(i) = \sum_j M_{ij} W_\sigma(j) \ , \tag{9}$$

where

$$M_{ij} = \frac{1}{\pi\sigma^2(r_{i+1}^2 - r_i^2)}\int_{r_i}^{r_{i+1}}\int_{r_j}^{r_{j+1}} r dr\, \overline{r} d\overline{r}\, e^{\frac{-(r-\overline{r})^2}{2\sigma^2}}\int_0^{2\pi} e^{-\frac{r\overline{r}}{\sigma^2}(1-\cos\overline{\theta})}\, d\overline{\theta} \ . \tag{10}$$

The integral over $\overline{\theta}$ can be reduced to a modified Bessel function, I_0. The double integration is done numerically, taking special care (along $i = j$) to provide the quadrature routine with an integrand that is not ill-behaved over the bin in question. The above equation assumes that the energy-deposition density does not vary significantly over the width of each bin.

4 Computer Codes for Offline Convolution

The following computer codes have been written in order to demonstrate the convolution technique presented in this paper.

- EGS4 User Code to create energy-deposition density data for small radial bins at various depths into the target. The code can be run with a pencil beam input, $\sigma = 0.0$. in order to generate the $W_\sigma(j)$ required by the off-line convolution scheme. It can also be run with a Gaussian incident beam, *e.g.,* $\sigma = 10$ or $\sigma = 100$ microns.
- A program that creates the Gaussian convolution matrix elements, M_{ij}, for a given set of beam σ, and a second program to check the $\sum_j M_{ij} = 1.0$ for any i-bin.
- A program to convolute $W_\sigma(j)$ and M_{ij} and produce a new output file for plotting the new $dE/E_0 dV$ (left ordinate) and the corresponding temperature rise, ΔT_p (°C/pulse) (right ordinate).

A common element with all of these codes is that they must have the same radial binning structure. In our example case (see Appendix 1), the target consists of 17 cylinders with radii of 1, 3, 5, 7, 10, 30, ..., 3000, 5000, 7000 10000 microns.

These codes are of a general enough nature to be of use in a variety of temperature-rise (and other) problems and can be downloaded from SLAC[2].

[2]The files are kept in `/afs/slac.stanford.edu/public/groups/egs4/Convolution` and can be obtained using anonymous ftp—*i.e.,* `ftp ftp.slac.stanford.edu` followed by `cd groups/egs4/Convolutions`).

4.1 EGS4 User Code to determine energy-deposition density, $W_\sigma(j)$

A general-purpose EGS4 User Code, called `ucRTZ_temp.mortran`, has recently been written at SLAC for a cylinder-slab geometry, with input read in from a `.data` file. This code is similar to the DOSRZ code by the National Research Council of Canada, which comes with the standard distribution of EGS4, but it is tailored for more general use other than dosimetry. For the problem of interest in this paper, another User Code was cloned from `ucRTZ_temp.mortran` and given the name `ucRTZ_temp_spot.mortran`, the only real difference being the addition of the capability of sampling the incident beam-spot size from a Gaussian distribution. Associated with this code is the input file `ucRTZ_temp_spot.data`, an example of which is provided in Appendix 1.

In the section below entitled Verification of Convolution Method we will use `ucRTZ_temp` to generate 10 computer runs, each with 1000 incident electrons, using two modes:

- Pencil-beam mode: With $\sigma = 0.0$ to create an output histogram, from the concatenation of ten runs, to be used as input for the convolution code.
- Direct-sampling mode: With σ set to either 10 or 100 microns to create an output histogram, again a concatenation of ten runs, that can be compared with the results of the convolution (at 10 and 100 microns).

4.2 Programs to create (and check) matrix elements, M_{ij}

After getting the basic set of histogram data—*i.e.,* ten separate runs of the User Code with different random number seeds—the next step is to create a set of matrices for each of the values of σ required in the problem. To facilitate this a program called `RTZ_mat.mortran` was created, documentation for which is contained within the code itself. Another code, `RTZ_matck.mortran`, has also been created to check that the $\sum_j M_{ij} = 1.0$ for any i-bin.

4.3 Program to convolute $W_\sigma(j)$ and M_{ij}

The program that performs the actual convolution is called `RTZ_temp_spot.mortran`. As described above, it requires the following two input files:

- `RTZ_temp_spot.data`
- `RTZ_mat.data`

The first file is a concatenation of EGS4 runs, ten in our example case, each created with the code `ucRTZ_temp_spot.mortran` and its input data file `ucRTZ_temp_spot.data`. The second file is a concatenation of the matrix output from one or more runs of `RTZ_mat.mortran` for each σ of interest. In our case, $\sigma = 3.16$, 10, 20, 30, 50, 100, 500, 1000, 2000 and 3000 microns. Note, however, that we will only make use of $\sigma = 10$ and 100 microns in our verification of the method, which is presented next.

5 Verification of Convolution Method

If a beam-spot size is directly sampled prior to each SHOWER call in EGS4 a lot of computer time is required in order to get adequate statistics, especially at high-energies, low cut-offs, and for thick materials. Hence, the reason for developing the convolution method in the first place. Nevertheless, the direct-sampling method itself provides us with a way to verify that the convolution technique works, provided that we limit the check to reasonably low-energy incident beams (note: a 1-GeV shower takes 100 times less time than does a 100-GeV shower).

Using the `ucRTZ_temp_spot` User Code, we have done this verification for an 8-r.l. thick copper target (1-cm radius) struck by 1-GeV electron beams. The target is broken up along Z into eight cylindrical slabs, each cylinder composed of 17 subcylinders, as indicated earlier (see Appendix 1).

Accordingly, this same radial structure was also employed with the `RTZ_temp_spot.mortran` and and `RTZ_mat.mortran` codes.

The results are shown in Figures 1 and 2 for beam sizes of 10 and 100 microns, respectively. The convoluted results (solid curves) are seen to be in excellant agreement with the directly-sampled results (histograms), at both the front (0-1 r.l.) and back (7-8 r.l.) of the target, thereby demonstrating that the convolution method works.

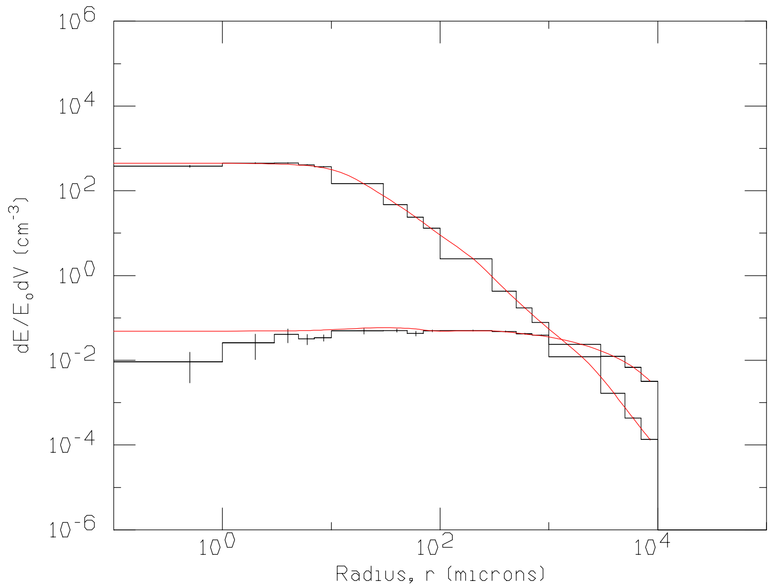

Figure 1: Comparision of direct-sampling (histogram) and convolution (solid curve) methods for a 1-GeV beam striking an 8-radiation length Cu cylinder: $\sigma_x = \sigma_y = 10$ microns.

6 Temperature-Rise Examples from the NLC

The beam parameters for the Next Linear Collider (NLC)[7] create a very serious problem with respect to the pulse temperature-rise in objects that the beam may inadvertantly strike. To illustrate this problem using the codes described in this paper, we considered a pencil-beam energy of 500 GeV impinging upon a 10-r.l. long Cu cylinder having a radius of 1 cm. The energy-deposition density and the corresponding pulse temperature rise are shown as a function of radius in Figures 3 through 5, for the beginning, middle and end of the target, respectively.

A full beam of 10^{12} electrons/pulse was used in these calculations and one sees that even in the first layer of the target (Figure 3), where the shower has yet to develop fully, the temperature rise on the pulse exceeds 20-million °C/p for the case of the pencil beam (histogram). In order to get the temperature down to something more reasonable, say 100 °C/p, the convolution curves tell us that the beam would have to be larger than 500 microns.

As the shower develops in the target, multiple scattering also leads to a lateral spread of the energy-deposition density. In Figures 4 and 5 we see that, indeed, the multiple scattering has some effect on reducing the temperature rise for incident beams with small emittance, but the shower multiplication is just too strong for large beams (*e.g.,* 500 microns), resulting in temperatures several orders of magnitude higher deep in the shower, relative to what they were near the surface.

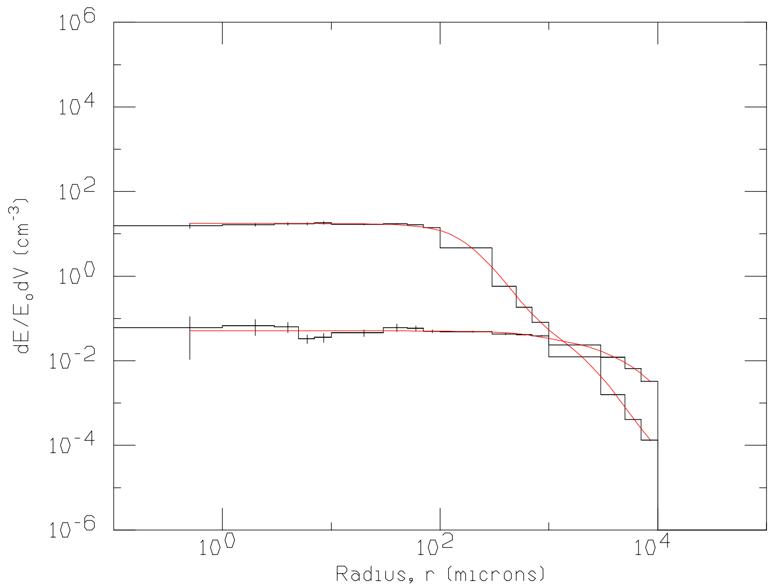

Figure 2: Comparision of direct-sampling (histogram) and convolution (solid curve) methods for a 1-GeV beam striking an 8-radiation length Cu cylinder: $\sigma_x = \sigma_y = 100$ microns.

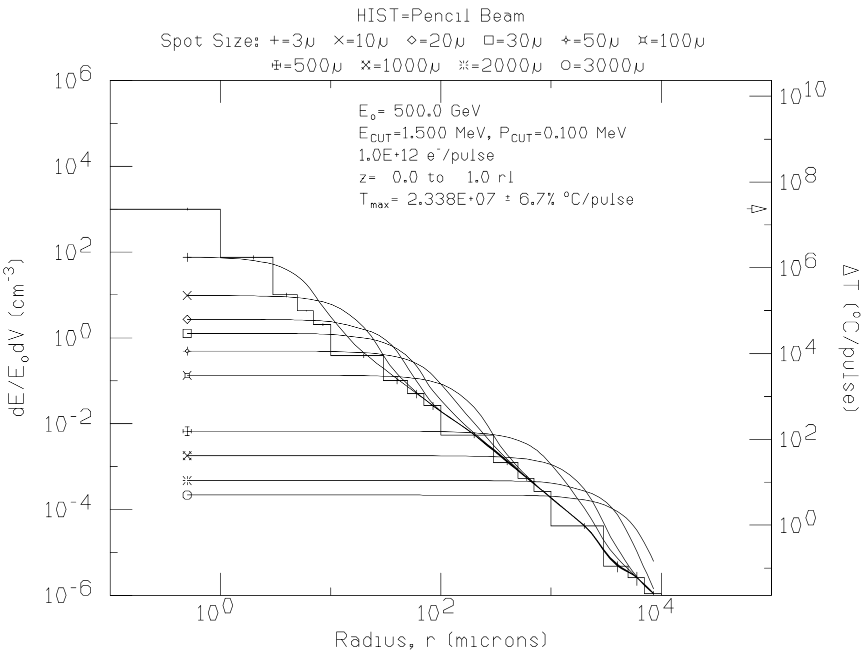

Figure 3: Energy-deposition density and temperature rise as a function of radius at the beginning of the target (Δz = 0-1 r.l.)

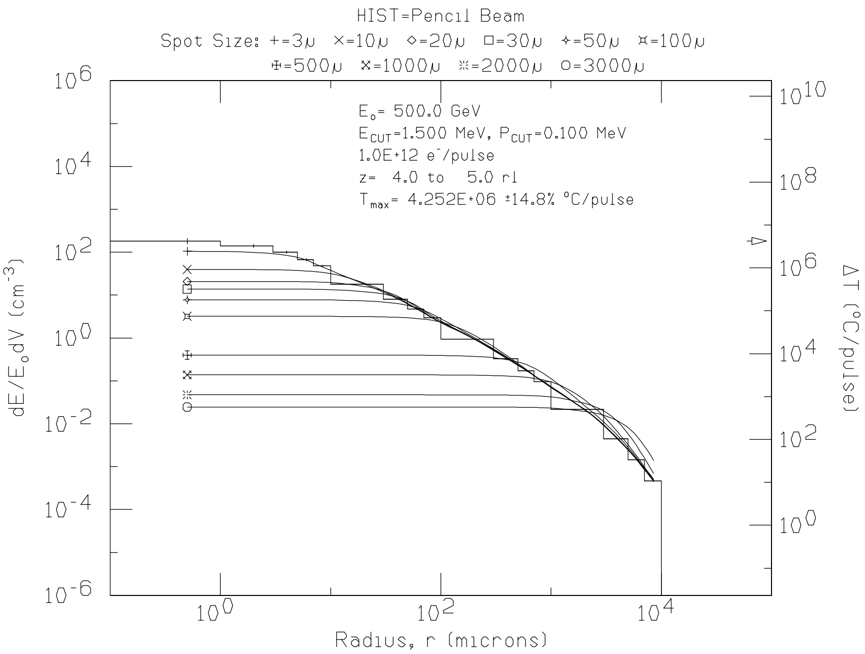

Figure 4: Energy-deposition density and temperature rise as a function of radius in the middle of the target (Δz = 4-5 r.l.)

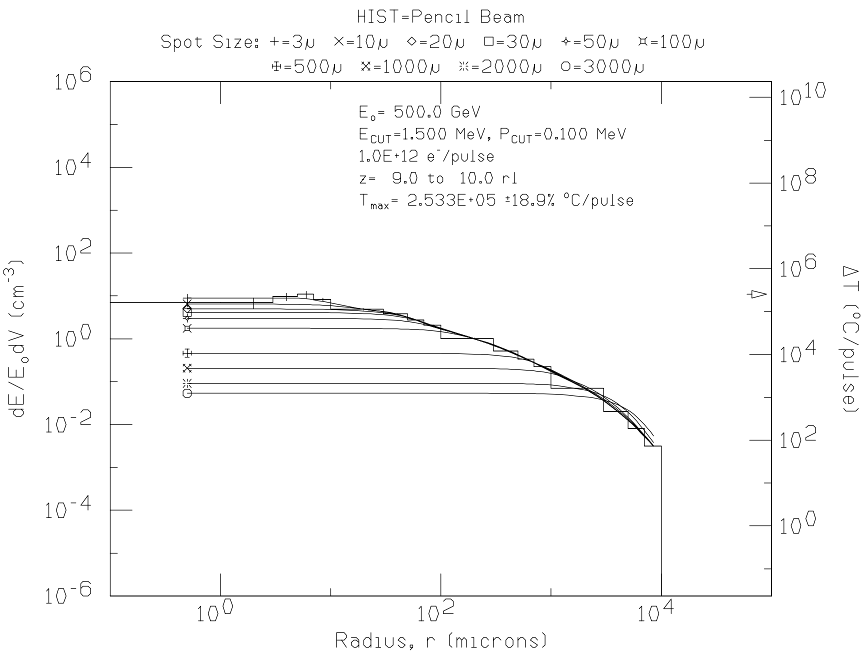

Figure 5: Energy-deposition density and temperature rise as a function of radius at the end of the target (Δz = 9-10 r.l.)

To show the dramatic effect of shower multiplicity on beams of all sizes, we plot in Figure 6 the maximum temperature rise as a function of depth, for the pencil beam and for the convolution curves of each of the ten incident beams considered. Also shown is the melting temperature of Cu (dashed line) and its stress limit (dotted line)[7].

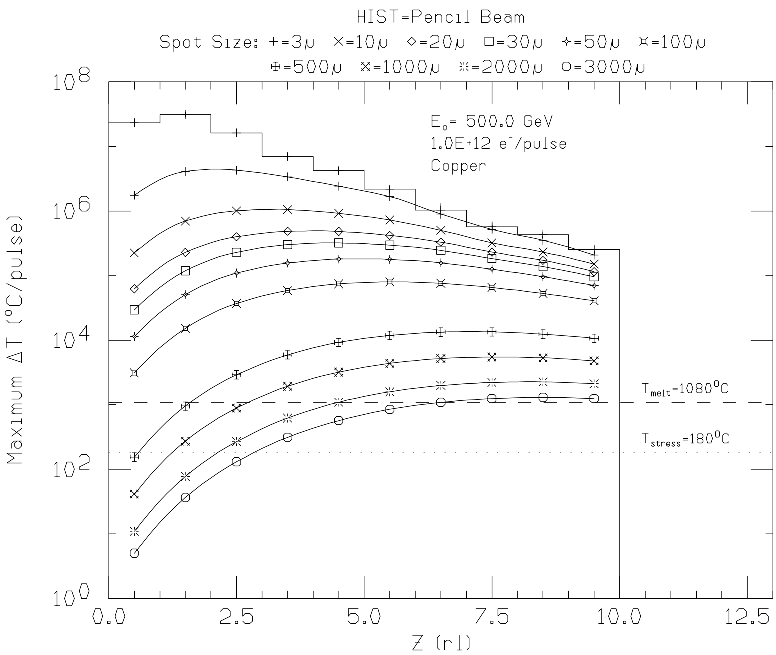

Figure 6: Pulse temperature rise vs. target depth for a 500 GeV beam in Cu

From this figure it becomes clear that even beams as large as 3000 microns could damage the copper target in a single pulse. The question then becomes: "Is there a material that is more suitable for full beams of the order of 10^{12} electrons/pulse?"

To answer this question we performed a similar analysis for Al and Be, with the results shown in Figures 7 and 8, respectively. The results show that aluminum and beryllium might be able to withstand a single pulse of the order of 1000 and 500 microns, respectively.

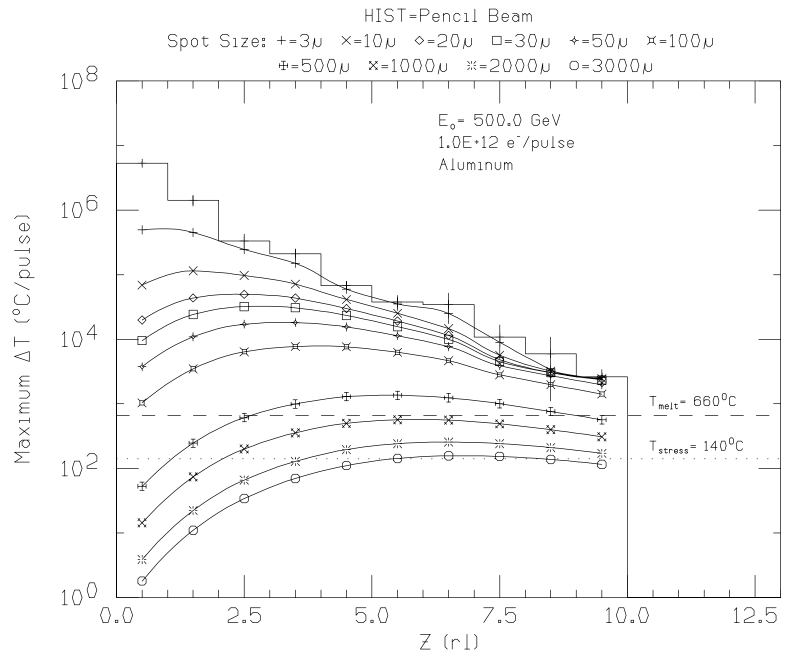

Figure 7: Pulse temperature rise vs. target depth for a 500 GeV beam in Al

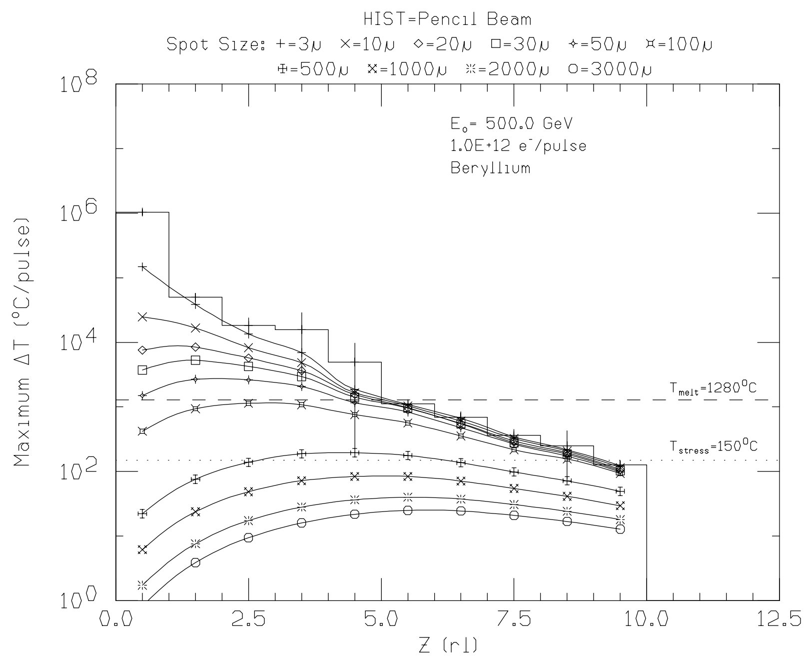

Figure 8: Pulse temperature rise vs. target depth for a 500 GeV beam in Be

7 Conclusion

Running thick-target EGS4 calculations at high energies can be costly in computer time. That is, the higher the energy the larger the shower multiplicity, implying that more particles must be followed until they reach their energy cutoffs. This, in turn, makes it difficult and time-consuming to study the temperature-rise in targets, where the size of the beam must also be incorporated into the Monte Carlo calculation in order to study real beams. Direct sampling over the beam size simply introduces yet another statistical variance into the results, requiring longer and longer jobs to be run.

In the study presented in this paper, we have created a convolution technique in which a reasonable set of computer runs, using a δ-function incident beam, can be used together with a predetermined set of beam matrices to obtain the temperature rise for a large number of beam sizes. We have demonstrated, at 1 GeV, that the results are consistent with the more laborious "direct sampling" approach, but the technique should be viable at any energy.

Also presented in this paper are some examples of the use of the convolution technique for temperature-rise studies for the Next Linear Collider, where the basic problem of very-small emittance beams of high-intensity and high-energy is shown to be formidable.

References

[1] W. R. Nelson, H. Hirayama and D. W. O. Rogers, "The EGS4 Code System", SLAC Report 265 (1985).

[2] H. DeStaebler, "Temperature Calculations for the Positron Target", SLAC Internal Report CN-21 (1980).

[3] H. DeStaebler, "Calculations for Positron Target Test in ESA" SLAC Internal Report CN-23 (1980).

[4] H. DeStaebler, "More Calculations for Positron Target Test in ESA" SLAC Internal Report CN-24 (1980).

[5] S. Ecklund and W. R. Nelson, "Energy Deposition and Thermal Heating in Materials Due to Low Emittance Electron Beams", SLAC Internal Report CN-135 (1981).

[6] R. J. Donahue and W. R. Nelson, "Alternative Positron-Target Design for Electron-Positron Colliders", LBL Internal Report LBL-30724 (1991) [and SLAC-PUB-5702 (1991)].

[7] "Zeroth-Order Design Report for the Next Linear Collider", SLAC Report 474 (May 1996).

Appendix 1

Representative example of the data-input file: `ucRTZ_temp_spot.data`

```
ucRTZ_temp_spot.data
         1                    NMED (I10)
CU                            MEDIA(J,1) (24A1)
       0.0       0.0          ECUTin,PCUTin (Kinetic) (MeV) (2F10.0)
        17         1         8 Imax,Jmax,Kmax (3I10)
    0.0001 I=1                CYRAD (cm) (F10.0)
    0.0003  =2
    0.0005  =3
    0.0007  =4
    0.0010  =5
    0.0030  =6
    0.0050  =7
    0.0070  =8
    0.0100  =9
    0.0300  =10
    0.0500  =11
    0.0700  =12
    0.1000  =13
    0.3000  =14
    0.5000  =15
    0.7000  =16
    1.0000  =17=Imax
       0.0  J=1=Jmax          THEPL (degrees) (F10.0) (no azimuthal)
       0.0  K=1               ZPL (r.l. here, hard coded to cm in program)
       1.0   =2
       2.0   =3
       3.0   =4
       4.0   =5
       5.0   =6
       6.0   =7
       7.0   =8=Kmax
       8.0   =9=Kmax+1
    1   17    1    1    1    8    1       0.0 CU
                                              blank card (required EOF)
       0.0       0.0       0.0 Xin,Yin,Zin (3F10.0)
         1         1         1 Iin,Jin,Kin (3I10)
       0.0       0.0       1.0 Uin,Vin,Win (3F10.0)
         1         1          IXX,JXX (2I10)
      1000    0.0100          Ncases,Sigma (I10,F10.0) (Sigma=0.0 implies pencil beam)
    1000.0        -1         0 EKEin(MeV),IQin,Isamp (F10.0,2I10)
    1    2    0    0          IBRDST,IPRDST,IBRSPL,NBRSPL (4I5)
    0    0    0    0       0.0 IPLC,IBCA,ILCA,IOLDTM,BLCMIN (4I5,F10.0)
```