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Applications of EGS4 on Leksell Gamma Unit

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Abstract

The Monte Carlo modelling of particle transport problems in medical radiation physics offers significant advantages over other techniques. Experiments can be performed without actually setting up the physical situation. Reliable results can be obtained without any unavoidable perturbations. In the present study, the EGS4 Monte Carlo code was applied to simulate the Leksell Gamma Knife - model B. The Leksell Gamma Knife is a surgical tool for brain lesions. It directs beams of Cobalt-60 radiation to a specific region to treat patients whose brain tumours or lesions are so deeply buried that conventional surgery is not possible.

The PRESTA (Parameter Reduced Electron-Step Transport Algorithm) version of the EGS4 (Electron Gamma Shower) computer code was employed. The EGS4 Monte Carlo code allows a more flexible geometrical simulation than other Monte Carlo code. The purpose of this report is to demonstrate how to apply the EGS4 Monte Carlo code in radiosurgical tool, the Leksell Gamma Knife, in order to calculate the radiation output factors and the dose distributions. The EGS4 user code allowed all the 201 Cobalt-60 sources to be simulated in a single session.

1 Methodology

1.1 Simulation of the Single-Beam Channel

The 201 radiation beams of the Gamma Knife unit were confined by the collimator system. Therefore, simulation of the collimator system defined the physics of the radiation beams. Primary photons exited from the Co-60 sources through the collimator system and interacted with the spherical water phantom.

The source body consists of the pre-collimator, the lead collimator, the exchangeable final collimator and the spherical water phantom. There are two available strategies in simulating the Co-60 source. First, as in the figure we simulated the whole single beam channel, including the cylindrical double encapsulated stainless steel source, pre-collimator, lead collimator and the interchangeable final collimator.

Second, each source was modelled by a cylinder without any source and capsule filtration. The beam was collimated by the internal diameters of the interchangeable final collimator mathematically. Radiation scattering due to the collimator system was ignored. Single beam dose profiles were generated by these two strategies. Because of larger source to target distance when compared with the source dimensions, no observable difference was obtained in the single-beam profiles. For simplicity, we employed the second strategy in simulating the 201 Co-60 sources.

1.2 EGS4 Monte Carlo Code

In the present work, PRESTA (Parameter Reduced Electron-Step Transport Algorithm) [1, 2] version of EGS4 (Electron Gamma Shower) Monte Carlo code was employed to calculate the dose outputs of the Leksell Gamma Knife - model B in a spherical water phantoms. The employment of

PRESTA in the simulation made the EGS4 Monte Carlo code select a suitable step length dynamically, which enables fast simulation while still providing accurate physics. Detailed descriptions on the structure of the EGS4 code can be found from Jenkins et al 1988[3]. The EGS4 code allows users to define a more flexible geometrical simulation than other Monte Carlo computer code. Leksell Gamma Knife involves a complicated geometrical layout and that is the reason why was the EGS4 code employed.

For the simulation, the patient's head was modelled by a water phantom 160 mm in diameter. Each one of the 201 sources located in the radiation unit is composed of 20 Co-60 pellets 1 mm in diameter and 1 mm in length. A total of 201 gamma beams exiting from the 201 Cobalt-60 sources were simulated. Each source was therefore modelled by a cylinder 1 mm in diameter and 20 mm in length.

The Co-60 sources are arranged in a sector of a hemispherical surface with a radius of 400 mm. They are distributed along five parallel circles separated from each other by an angle of 7.5°[4].

The 201 radiation beams passed through the opening of the collimators to the target point. The diameters of the radiation beams at the focus were confined by the size of collimators which are 4, 8, 14 and 18 mm. The measured internal diameters of the 4 mm collimator were 2.14 mm and 2.66 mm. For the 8 mm collimator, they were 3.92 mm and 5.00 mm. For the 14 mm collimator, they were 6.52 mm and 8.56 mm. For the 18 mm collimator, they were 8.26 mm and 10.88 mm.

1.3 Coordinates of the 201 Co-60 sources

For simplicity in calculations, we needed to treat the unit centre point as (0,0,0) instead of (100,100,100). In Fig. 1, the angle θ was defined on every rows of collimators inclined to the z-plane. The values θ are given in Table 1.

By considering the spacing between collimators on each row in Figure 2, we can calculate an angle δ between collimators on each row. The values of spacing S between collimators on each row were obtained based on physical measurement on the collimator helmets. The values S are given in Table 2.

Therefore, the values of angle δ can be obtained using the following equation.

$$\delta = \frac{S}{225 \times \cos\theta}$$

The coordinates in mm (x, y, z) of collimators on the surface of the helmet are obtained by the following equations.

$$\begin{cases} x = 225 \times \cos\theta \times \sin\delta \\ y = 225 \times \cos\theta \times \cos\delta \\ z = -225 \times \sin\theta \end{cases}$$

Consider the source to focus distance to be 400 mm. The coordinates in mm (x, y, z) of the 201 Co-60 sources can be obtained by the following equations.

$$\begin{cases} x = 400 \times \cos\theta \times \sin\delta \\ y = 400 \times \cos\theta \times \cos\delta \\ z = -400 \times \sin\theta \end{cases}$$

1.3.1 Calculation of the target points

Consider a point P(x_1,y_1,z_1) at the central axis of the cylindrical Co-60 source. A photon exits from the source and targets to the spherical water phantom. The direction cosine of the line joining the point P and the unit centre point (0, 0, 0) is

$$\left(\frac{x_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}, \frac{y_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}, \frac{z_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}} \right).$$

Define an arbitrary point M(x,y,z) on the plane perpendicular to the incident ray. Then, the equation of the plane is

$$(x, y, z) \cdot \left(\frac{x_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}, \frac{y_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}, \frac{z_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}} \right) = 0.$$

Or,

$$(xx_1 + yy_1 + zz_1) = 0. \tag{1}$$

The equation of the target sphere with radius R is: $x^2 + y^2 + z^2 = R^2$.
The equation becomes

$$x^2 + y^2 + z^2 \leq R^2, \tag{2}$$

if any point (x,y,z) is inside the volume of the target sphere.
From the equation [1], we write:

$$z = -\left(\frac{xx_1 + yy_1}{z_1} \right) \tag{3}$$

and substitute z into the equation [2].
Now, the equation [2] becomes:

$$x^2 + y^2 + \left(\frac{xx_1 + yy_1}{z_1} \right)^2 \leq R^2.$$

Then,

$$x^2 \left(1 + \frac{x_1^2}{z_1^2} \right) + y^2 \left(1 + \frac{y_1^2}{z_1^2} \right) + \frac{2xyx_1y_1}{z_1^2} \leq R^2.$$

We choose random numbers x and y in mm, so that

$$\begin{cases} -R \leq x \leq R \\ -R \leq y \leq R \end{cases}$$

and z can be determined by using the equation [3].

1.4 Determination of the radius R of the target sphere

The maximum radius R(mm) should include the region of geometrical penumbra. Therefore, by considering a photon exiting from the circumference and the bottom of the cylindrical source, the value of R can be found (Figure 3).

In trigonometry, the addition formula of the function tangent is:

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta}$$

In Figure 3, the value of α and β are given below,

$$\alpha = \tan^{-1} \left(\frac{r_{out} + 0.5}{390 - 165} \right), \; \beta = \tan^{-1} \left(\frac{0.5}{390} \right) \text{ and } \tan(\alpha - \beta) = \frac{R}{\sqrt{0.5^2 + 390^2}}.$$

Therefore,

$$R = \sqrt{0.5^2 + 390^2} \left[\frac{\left(\frac{r_{out}+0.5}{390-165} \right) - \left(\frac{0.5}{390} \right)}{1 + \left(\frac{r_{out}+0.5}{390-165} \right) \cdot \left(\frac{0.5}{390} \right)} \right],$$

where R_{out} is the internal radius of the collimator at the exiting end.

1.5 Confining a photon passing through the collimator

In Figure 4, define a point A at the central axis and at the exiting end of collimator, another point B at the central axis and at the incident end of collimator. Then, any points exiting from within the cylindrical source try to point at the target (x_t, y_t, z_t) and pass through the collimator. At the incident end and exiting end of the collimator, we define a point C and a point D at the interfaces. Therefore, if the distance BC is less than the radius of the incident end of the collimator and the distance AD is less than the radius of the exiting end of the collimator, then the emerging photon is able to pass through the whole collimator. The coordinate of point C and point D can be found as follow.

Direction cosine of OP is:

$$\left(\frac{x_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \frac{y_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \frac{z_1}{\sqrt{x_1^2+y_1^2+z_1^2}}\right).$$

Then, the equation of the plane containing point B is:

$$(x-x_B, y-y_B, z-z_B)\cdot\left(\frac{x_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \frac{y_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \frac{z_1}{\sqrt{x_1^2+y_1^2+z_1^2}}\right) = 0.$$

Then, we have

$$(x-x_B)\cdot x_1 + (y-y_B)\cdot y_1 + (z-Z_B)\cdot z_1 = 0. \tag{4}$$

Consider the equation of the photon exiting from Q to the target (x_t, y_t, z_t):

$$\frac{x-x_{in}}{x_{in}-x_t} = \frac{y-y_{in}}{y_{in}-y_t} = \frac{z-z_{in}}{z_{in}-z_t}.$$

Therefore,

$$x = \left(\frac{y-y_{in}}{y_m-y_t}\right)\cdot(x_{in}-x)+x_{in}, \tag{5}$$

and

$$z = \left(\frac{y-y_{in}}{y_m-y_t}\right)\cdot(z_{in}-z)+z_{in}, \tag{6}$$

Substitute the equations [5] and [6] into [4]. We get the result of y as follow.

$$y = \frac{(z_B-z_{in})+(x_B-x_{in})\cdot x_1 + y_B y_1 + y_{in}\cdot\left(x_1\cdot\left(\frac{x_{in}-x_t}{y_{in}-y_t}\right)+z_1\cdot\left(\frac{z_{in}-z_t}{y_{in}-y_t}\right)\right)}{\left(x_1\cdot\left(\frac{x_{in}-x_t}{y_{in}-y_t}\right)+z_1\cdot\left(\frac{z_{in}-z_t}{y_{in}-y_t}\right)\right)+y_1}$$

Therefore, the point C can be found. The point D can also be found in a similar way.

1.6 Obtaining the coordinates of the point A and B

In Figure 5, consider the equation of line OP:

$$\frac{x_1}{x_A} = \frac{y_1}{y_A} = \frac{z_1}{z_A} \tag{7}$$

and the equation of sphere A:

$$x^2+y^2+z^2 = 165^2 \tag{8}$$

Substitute the equations [7] into [8].

We obtain the coordinate of point A.

$$z_A - \frac{-165}{\sqrt{\frac{x_1^2}{z_1^2}+\frac{y_1^2}{z_1^2}+1}}$$

and

$$x_A = \frac{z_A x_1}{z_1},\ y_A = \frac{z_A y_1}{z_1}$$

With the equation of sphere B:$x^2 + y^2 + z^2 = 225^2$. The coordinate of point B can be found in a similar way.

1.7 Select points within the cylindrical source

We needed to randomly select a point Q within the cylindrical source (Figure 6). At first, a randomly selected point R was found based on the distance ratio OR and OP.

That is,

$$R(x_2, y_2, z_2) = \frac{|OR|}{|OP|} \cdot P(x_1, y_1, z_1).$$

With the direction cosine of OR:

$$\left(\frac{x_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}}, \frac{y_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}}, \frac{z_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}} \right),$$

equation of the plane containing R becomes:

$$(x - x_2, y - y_2, z - z_2) \cdot \left(\frac{x_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}}, \frac{y_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}}, \frac{z_2}{\sqrt{x_2^2 + y_2^2 + z_2^2}} \right) = 0$$

Or,

$$(x - x_2) \cdot x_2 + (y - y_2) \cdot y_2 + (z - z_2) \cdot z_2 = 0. \tag{9}$$

Consider the equation of solid sphere with the point R as a centre:

$$(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 \leq 0.5^2, \tag{10}$$

where 0.5 is the radius of the cylindrical source.

Solve the equations [9] and [10]. We have,

$$z = \frac{(x_2 - x) \cdot x_2 + (y_2 - y) \cdot y_2}{z_2} + z_2$$

and the values of x and y were selected from random numbers as follow.

$$\begin{cases} -0.5 \leq y - y_2 \leq 0.5 \\ -0.5 \leq x - x_2 \leq 0.5 \end{cases}$$

The coordinate of point Q can be found. Therefore, any photons exiting within the cylindrical source can find a target coordinate on the plane perpendicular to the direction of incident photon.

2 Discussion and Conclusions

Experimental determination of the output factors are difficult due to the extremely narrow beams for which the dose is determined. In the simulation, a spherical probe with a radius of 1 mm was utilized at the centre of the spherical water phantom, 160 mm in diameter. Our simulation confirmed the new 4 mm output factor[5, 6] provided by ELEKTA (Manufacturer of Leksell Gamma Knife). EGS4 Monte Carlo technique was also employed to calculate the dose distribution along the x, y and z axes when a single shot with the opening of all 201 sources was delivered at the centre of a simulated water phantom. Different collimator helmets of 4, 8, 14 and 18 mm were studied. Good agreements were obtained between the results of the planning system and the Monte Carlo. Small discrepancies were observed along the z-axis of the 8 mm collimator helmet[7]. Furthermore, the change in dose distributions can also be predicted after applying some plugged patterns at the collimator helmets[8]. Owning to the energy dependency of radiographic films, accurate measurement results may not be obtained easily[9]. Therefore, the applications of EGS4 Monte Carlo technique is important and should be included as a part of quality assurance program for the Gamma Knife radiosurgery.

Table 1 Row A to E inclined to the z-plane by an angle θ.

Row	θ
A	6.0°
B	13.5°
C	21.0°
D	28.5°
E	36.0°

Table 2 The spacing S between collimators within the row from A to E.

Row	**S(mm)**
A	30.00
B	30.55
C	33.00
D	31.06
E	31.77

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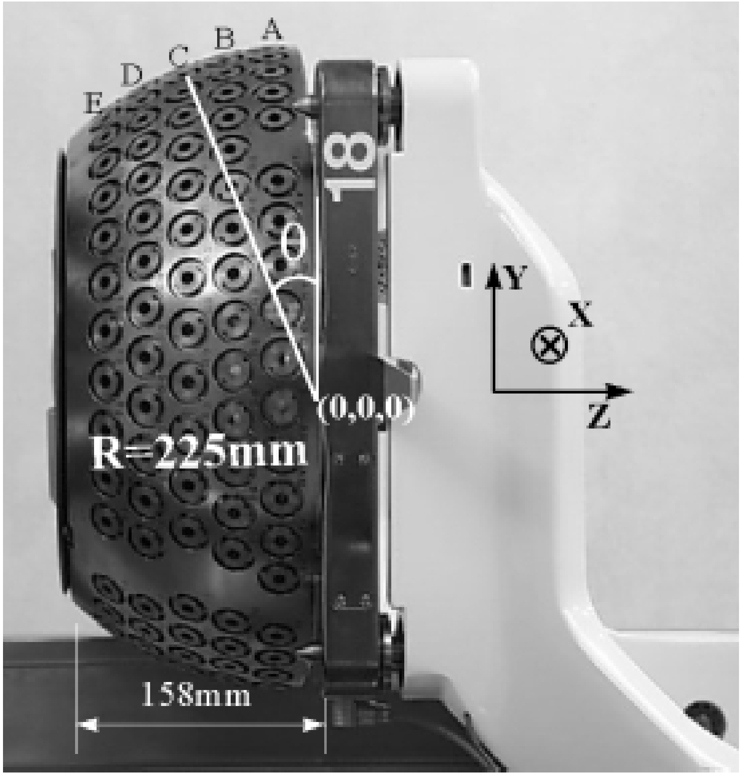

Figure 1 An angle θ inclined to the z-plane of the collimator helmet.

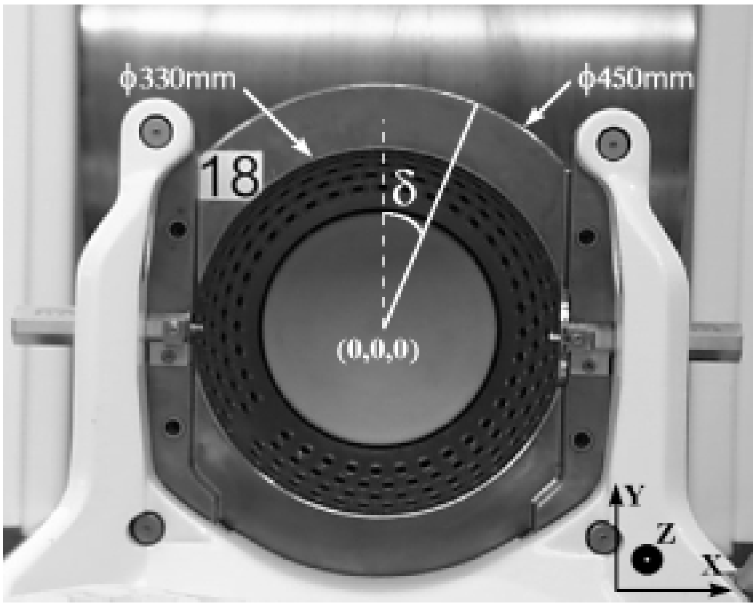

Figure 2 An angle δ inclined to x-plane of the collimator helmet

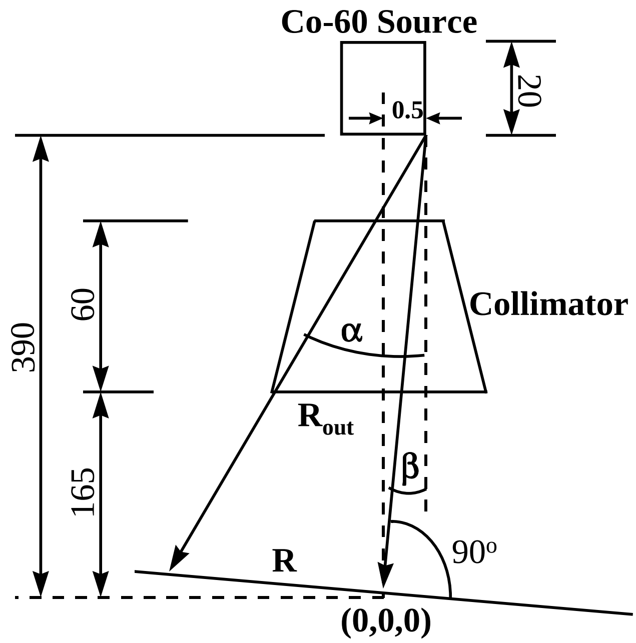

Figure 3 The calculation of a maximum radius R at the target plane.

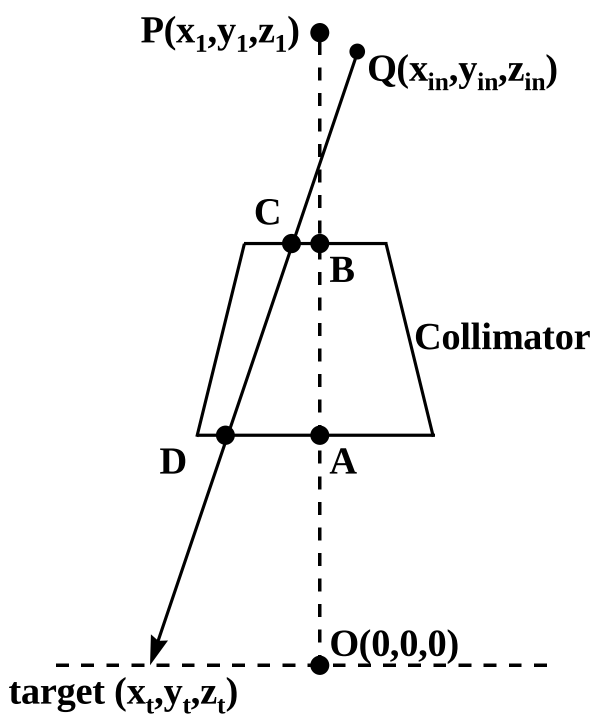

Figure 4 The calculation of coordinates of the points C and D interacting the collimator.

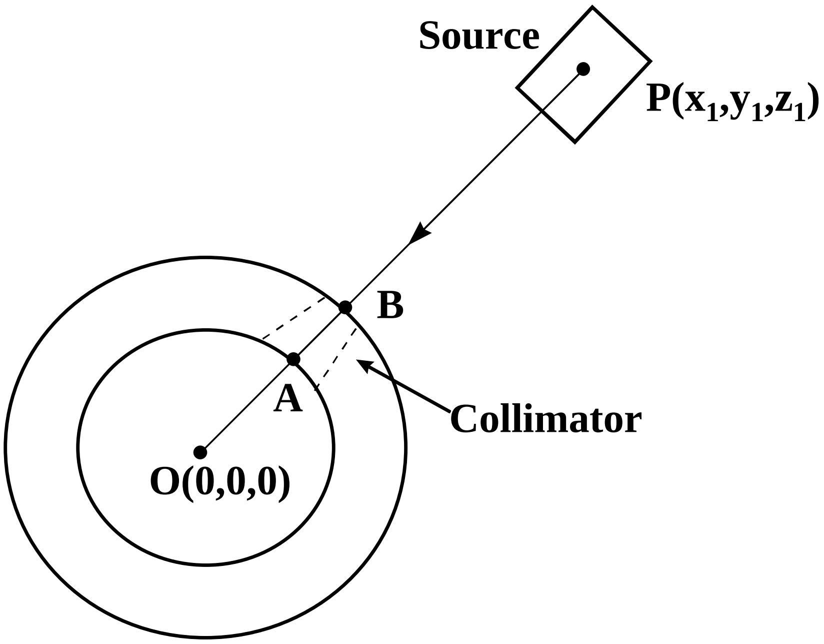

Figure 5 The calculation of coordinates of points A and B at the central axis of the collimator.

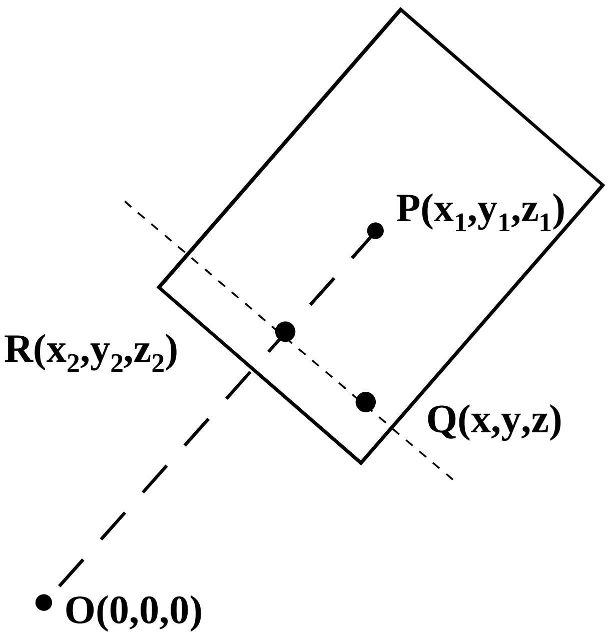

Figure 6 The calculation of random selected points within the volume of the cylindrical